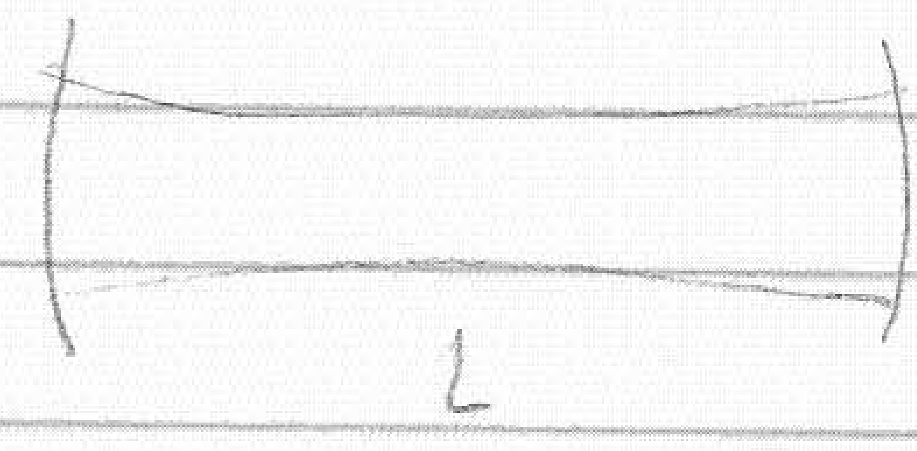
Example: compare a near-planar resonator w/ confocal

symmetric cavity $R_1 = R_2 = R$

separation $L << R$



what is spot size at end mirror and waist?

$$R(z) = z\left(1 + \left(\frac{z_R}{z}\right)^2\right) \qquad z = \text{dist to waist} = L/2$$

$$R = \frac{L}{2}\left(1 + 4\frac{z_R^2}{L^2}\right)$$

$$z_R = \frac{L}{2}\left(\frac{2R}{L} - 1\right)^{1/2} \qquad R >> L \text{ so } z_R \approx \sqrt{\frac{RL}{2}}$$

and $w_0 = \left(\frac{\lambda z_R}{\pi}\right)^{1/2}$

for $L = 1\,m$ $R = 8\,m$ $\rightarrow$ $w_0 \approx 0.57\,mm$

$\lambda = 514\,nm$ (Ar^+ laser)

At end mirrors?

$$w(z) = w_0\sqrt{1 + (z/z_R)^2}$$

$$= w_0\sqrt{1 + \left(\frac{L}{2}\sqrt{\frac{2}{RL}}\right)^2} = w_0\sqrt{1 + \frac{L}{2R}}$$

in this case $w_1 = w_0 \cdot (1 \tfrac{1}{32})$ ~ constant mode size.

Compare to confocal of same L:

$R_1 = R_2 = L = 1\,m$

$$z_R = \frac{L}{2}(2-1)^{1/2} = \frac{L}{2} \rightarrow w_0 = \left(\frac{\lambda L}{2\pi}\right)^{1/2} = 0.29\,mm$$

$$w_1 = \sqrt{2}\,w_0 = 0.4\,mm$$

ABCD for quadratic index / gain profiles

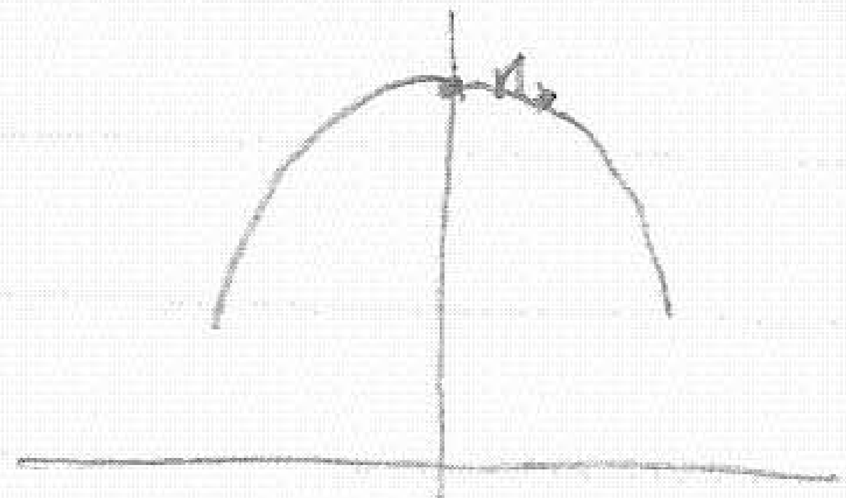
ideal lens converts plane wavefront to a section of a spherical wave front:

$$E_{out}(r) = E_{in}(r)\, e^{-ikr^2/2f}$$

$\therefore$ we are putting a parabolic phase shift on the beam.

We could do the same with a gradient-index medium:

$$n(r) = n_0\left(1 - \frac{k_2}{2k} r^2\right)$$



k_2 = constant.

then propagation phase is: $e^{ik_0 n_0 L}\, e^{-k_0 n_0 k_2 r^2/2\, L}$

$$f = k_2 L$$

- lenses of this type can be manufactured by diffusing impurities
- thermal profile $\rightarrow$ parabolic
  - $\rightarrow$ gradient index lensing.
- can also apply to gradient in gain.
- gradient-index optical fiber

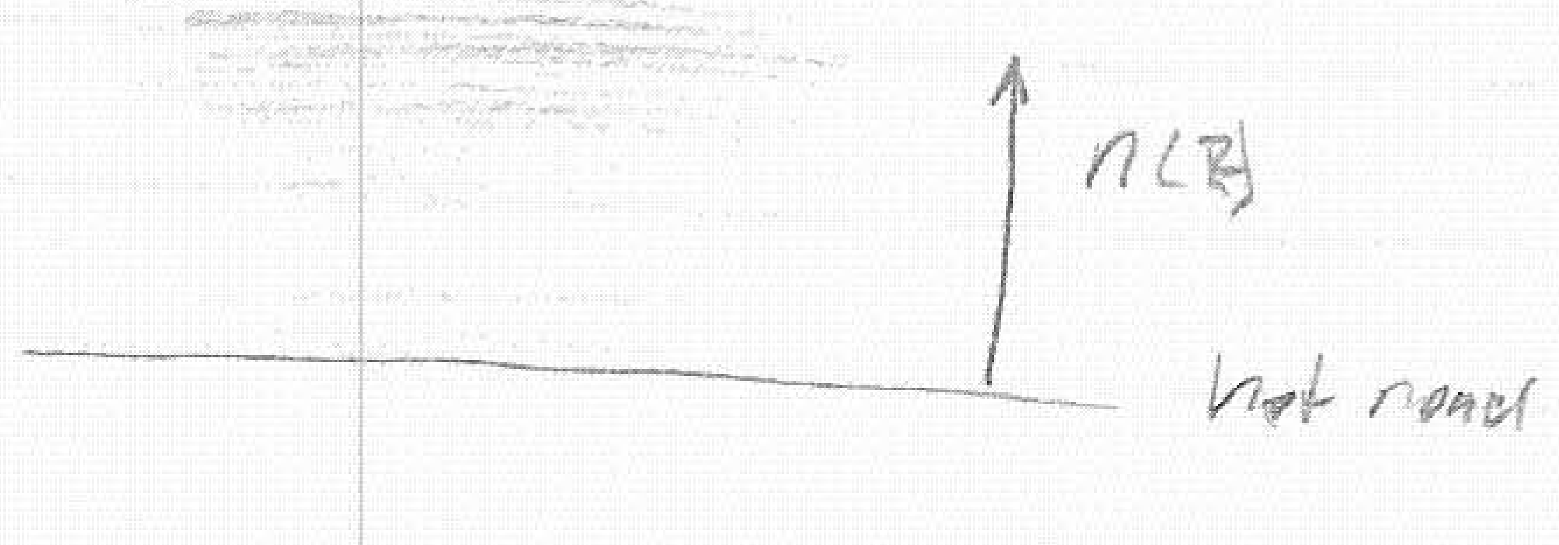
How do rays propagate through this medium?

ray equation: $\frac{d}{ds}\left(n \frac{d\vec{r}}{ds}\right) = \nabla n$ s = distance along ray.

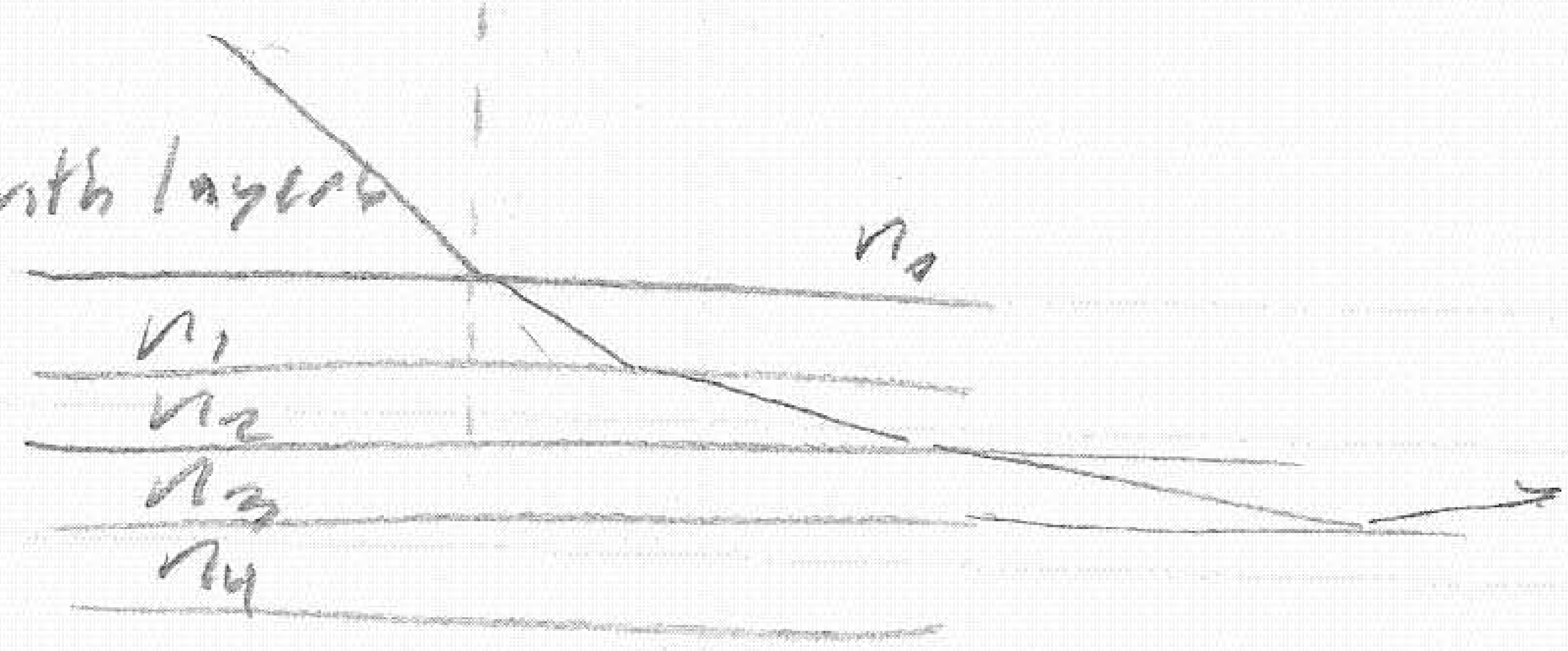
the gradient in $n(\vec{r})$ changes ray direction

example: mirage

low density, low n near road.



model with layers



$n_0 > n_1 > n_2 > n_3 > n_4$ - - -

ray bends away from normal

$n_i \sin\theta_i$ constant

eventually reach critical angle for TIR

$\theta_j = 90°$ $\quad \sin\theta_j = 1$

$n_0 \sin\theta_0 = n_j$

Same principle for gradient index:

turning pt. of wave where $n_0 \sin\theta_0 = n(r_t)$

$\therefore$ expect guiding

For parabolic index, paraxial $\quad s \sim z$

ray eqn $\rightarrow$ $\quad \dfrac{d^2 r}{dz^2} + \dfrac{k_2}{k} r = 0 \quad$ osc. eqn.

solution $\quad r(z) = \cos\left(\sqrt{\dfrac{k_2}{k}}\, z\right) r_0 + \sqrt{\dfrac{k}{k_2}} \sin\left(\sqrt{\dfrac{k_2}{k}}\, z\right) r_0'$

$r'(z) = -\sqrt{\dfrac{k_2}{k}} \sin\left(\sqrt{\dfrac{k_2}{k}}\, z\right) r_0 + \cos\left(\sqrt{\dfrac{k_2}{k}}\, z\right) r_0'$

$\rightarrow$ ABCD form.

note period is $\left(\sqrt{k/k_2}\right) 2\pi$

- This ABCD also works for Gaussian beam propagation.
- if $k_2 < 0$ (index increases with r)

$\cos \rightarrow \cosh \quad \sin \rightarrow i \sinh \quad$ beam defocuses.

- quadratic gain profile:

$$k(r) = k \pm i\left(\alpha_0 - \frac{1}{2}\alpha_2 r^2\right)$$

$+ \rightarrow$ gain

$- \rightarrow$ loss

for $k_2 r^2 << k \quad \rightarrow \quad k_2 = i\alpha_2$

can use same ABCD

$\rightarrow$ "gain guiding"

There is a stable mode size (no change during propagation)

- mode matching.

$$q(z) = \frac{q_0 \cos\beta z + \frac{1}{\beta}\sin\beta z}{-q_0\beta\sin\beta z + \cos\beta z} = q_0 \qquad \beta = \sqrt{\frac{k_2}{k}}$$

$\hookrightarrow$ at stable mode

$$q_0 + \frac{1}{\beta}\tan\beta z = -q_0^2\beta\tan\beta z + q_0$$

if $\frac{1}{\beta} + q_0^2\beta = 0$ no z dependence

for quad index $\frac{1}{q} = \frac{1}{R} - i\frac{1}{z_R}$

if $R = \infty \quad q_0^2 = -z_R^2$

$\rightarrow z_R^2 = 1/\beta^2 = k/k_2$

for gain media $\rightarrow$ finite radius, stable mode size.