Homework 2 due Thursday, 30 January 2014
PHGN 570: Physical and Fourier Optics

1. Show that the self-convolution of the function $\operatorname{sinc}(at/2)$ is the same function multiplied by a constant.
2. A "wavelet transform" is the Fourier transform of a function whose spectrum changes with time, and consists of a representation of the Fourier transform measured during an interval δt as a function of time. It is often used in speech and music analysis. With the help of the convolution theorem, show that δt and the frequency resolution $\delta\omega$ of the wavelet transform are related by $\delta t \cdot \delta\omega \approx 2\pi$.

3. Consider an LSI system with an impulse response $h(x) = \exp(-x)$ for x>0, and $h(x) = 0$ for x<0. The input pulse to the system is $f(x) = rect(10x - 0.5)$, where $rect(x) = 1$ for |x|< ½, and 0 otherwise. Calculate and the output signal $g(x) = h(x) \otimes f(x)$ in several different ways. For each case, plot the outputs to make sure you get the same results in each case.
   a. Analytically (without using Mathematica for symbolic manipulation), by doing the direct convolution integral.
   b. Analytically, by using the convolution theorem: $g(x) = \Im^{-1}\{F(k)H(k)\}$.
   c. Analytically (in Mathematica). The impulse response function can be represented in Mathematica by $Exp[-x]UnitStep[x]$.The rect(x) function can be written as $UnitStep[\frac{1}{2} - x]UnitStep[\frac{1}{2} + x]$ or $\mathrm{UnitBox}[x]$. Use Integrate[ ] to do the calculation.
   d. Numerically, using ListConvolve[ ]. You'll have to read the help notes to understand how it works. Note that the output list is normally shorter than the input lists, unless you pad the ends with zeroes.
4. Spectral interferogram. Spectral interferometry does not require coherent light. Show that if the input spectrum is $E(\omega) = A(\omega)\exp[i\phi_r(\omega)]$, where $\phi_r(\omega)$ is a random spectral phase, there is still an interferogram when the spectral intensity of this field plus a time-shifted copy is measured.
5. Spectral interferometry can be used to characterize nonlinear changes to a pulse. These often with change the spectral bandwidth.
   a. Calculate a general expression for the spectral interferogram of two Gaussian pulses with amplitudes A_1 and A_2 and pulse durations t_1 and t_2. The carrier frequency for both pulses is ω_0 and the time delay between the pulses is τ. Starting from the time domain calculate the spectral intensity. If you use Mathematica to perform the transforms and calculations, write the final result in a simplified form.

   To make plots, assume the central wavelength is λ_0 = 600nm and the duration of the first pulse is t_1 = 100fs. Plot the spectral interferogram (vs ω measured in rad/fs) for the following cases:
   b. t_2 = 100fs. Set $A_1 = A_2$ so that each spectrum is normalized to 1. Choose the time delay to get approximately 10 fringes across the spectral width.

c. Same parameters as (b), but pick a2 so that the energy of pulse 2 is only 1% of that of pulse 1.
d. Return to $A_1 = A_2$, but now let t2 = t1/sqrt[3].
e. Return to the conditions of (b), but multiply the second spectral field by the quadratic phase factor $\exp\left[i\phi_2\left(\omega-\omega_0\right)^2\right]$, where ϕ_2 is a constant with the value $\phi_2 = 3\times10^4\, fs^2$.