

## Interaction of atoms with radiation

outline: BB radiation + resonators

spectral energy density

summary of ideal periodic field quantization

spontaneous emission (intro)

absorption and stimulated emission

## EM waves - summary.

from Maxwell eqns, wave eqn for E field (3-D)

$$\nabla^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad n = \text{refractive index}$$

1D version: propagation along z, no variation in x, y

$$\vec{E}(\vec{r}, t) \rightarrow \vec{E}(z, t), \quad \nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$$

$$\nabla^2 \vec{E} \rightarrow \partial_z^2 \vec{E}$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

with no free charges  $\nabla \cdot \vec{E} = 0 \rightarrow \frac{\partial E_z}{\partial z} = 0$  here.

$$\therefore E_z = \text{constant (DC, non propagating)} \frac{\partial^2}{\partial z^2} \rightarrow 0$$

only  $E_x, E_y$  components. (polarization)

choose  $E_x$

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{n^2}{c^2} \frac{\partial^2 E_x}{\partial t^2} = 0$$

separable solution  $E_x(z, t) = f(z)g(t)$

evaluate derivatives, divide by  $f(z)g(t)$

$$\rightarrow \frac{1}{f} \frac{\partial^2 f}{\partial z^2} - \frac{n^2}{c^2} \frac{1}{g} \frac{\partial^2 g}{\partial t^2} = 0$$

each term = constant,

$$\frac{1}{f} \frac{d^2 f}{dz^2} = \alpha \quad \text{try oscillating solutions } f \sim A e^{\pm i k z}$$

$$\rightarrow -k^2 = \alpha$$

$$\frac{n^2}{c^2} \frac{1}{g} \frac{d^2 g}{dt^2} = \alpha \quad \text{also try } g \sim B e^{\pm i \omega t}$$

$$\rightarrow -\frac{n^2 \omega^2}{c^2} = \alpha = -k^2$$

$$\therefore k = \pm \frac{n \omega}{c}$$

$$\text{Full solution } E_x = A e^{\pm i k z \pm i \omega t}$$

notes

$\omega$  = angular frequency =  $2\pi \nu$   
= constant through diff't media

$$k = \text{wave number} = \frac{2\pi}{\lambda} = n \cdot \frac{\omega}{c}$$

define  $k_0 \equiv \omega/c$   $k = n k_0 = \text{wave number in medium}$   
 $\lambda = \lambda_0/n$  wavelength in medium

relative sign  $\text{medium} \rightarrow \text{vacuum}$

$$kz \mp \omega t \equiv \phi(z, t) = \text{phase, position w/in } \sim$$

$\rightarrow - \rightarrow$  wave moves in  $+z$  direction

$+ \rightarrow$  wave moves in  $-z$  direction.

We will use convention with  $e^{\pm i k z - i \omega t}$   
(Svelto uses  $e^{-i k z + i \omega t}$ )

$$c/n = \text{phase velocity} = v_{ph}$$

E-field is real, not complex

We use complex, phasor notation for convenience.

- take real part at the end.

can write  $A e^{i(kz - \omega t + \phi_0)}$  then take real part.  
or  $A e^{i(kz - \omega t + \phi_0)} + \text{c.c.}$

can absorb phase into amplitude coeff.

$$A e^{i\phi_0} \rightarrow A \text{ (complex)}$$

$\downarrow$  real  $\downarrow$  real

or can use cosines + sines:

$$A \cos(kz - \omega t + \phi_0)$$

$$\text{or } A \cos(kz - \omega t) + B \sin(kz - \omega t)$$

There is also a wave eqn for B field, w/ same solutions.  
we will usually just work w/ E-field.

Resonator - 1D, vacuum, perfectly conducting ends (mirrors)

assume  $E(0) = E(L_x) = 0$

- E-component tangent to conductor = 0

$$E_x(z, t) = (A_+ e^{+ik_z z} + A_- e^{-ik_z z}) e^{-i\omega t}$$

= superposition of +ve, -ve propagating waves.

for  $E_x = 0$  at  $z=0 \rightarrow A_+ = -A_-$  or write

$$E_x(z, t) = A \sin k_z z e^{-i\omega t}$$

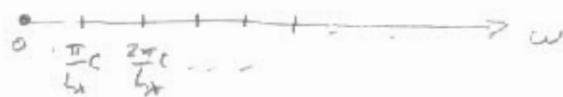
for  $E_x = 0$  at  $z=L_x \rightarrow k_z L_x = l\pi \quad l = 1, 2, 3, \dots$

or  $k_z = \frac{l\pi}{L_x}$  or  $l \frac{\lambda}{2} = L_x$  integer # half wavelengths.

if these are the allowed  $\lambda$ 's what are the frequencies?

$$\frac{l\pi}{L_x} = \frac{\omega_l}{c}$$

these are the resonant frequencies



these are the longitudinal modes of the resonator.

equal spacing:

$$\Delta\omega = \frac{\pi c}{L_x} \quad \text{or} \quad \Delta\nu = \frac{c}{2L_x} = \frac{1}{\tau_{\text{round trip}}}$$