Interaction of atoms with radiation

outline: BB radiation + resonators

Spectral energy density

Summary of ideas behind field quantization

Spontaneous emission (intro)

absorption and stim. emission

EM waves - summary.

From Maxwell eqns, wave eqn for E field (3-D)

$$\nabla^2 \vec{E} - \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad n = \text{refractive index}$$

1 D version: propagation along z, no variation in x, y

$$\vec{E}(\vec{r},t) \to \vec{E}(z,t), \quad \nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$$

$$\nabla^2 \vec{E} \to \partial_z^2 \vec{E}$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} - \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

with no free charges $\nabla \cdot \vec{E} = 0 \to \frac{\partial E_z}{\partial z} = 0$ here.

$\therefore E_z =$ constant (DC, non propagating)

$\to 0$

only E_x, E_y components (polarization)

choose E_x

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{n^2}{c^2}\frac{\partial^2 E_x}{\partial t^2} = 0$$

separable solution $E_x(z,t) = f(z)g(t)$

evaluate derivatives, divide by $f(z)g(t)$

$$\to \frac{1}{f}\frac{\partial^2 f}{\partial z^2} - \frac{n^2}{c^2}\frac{1}{g}\frac{\partial^2 g}{\partial t^2} = 0$$

each term = constant.

$$\frac{1}{f}\frac{\partial^2 f}{\partial z^2} = \alpha$$ try oscillating solutions $f \sim A e^{\pm i k z}$

$$\rightarrow -k^2 = \alpha$$

$$\frac{n^2}{c^2}\frac{1}{g}\frac{\partial^2 g}{\partial t^2} = \alpha$$ also try $g \sim B e^{\pm i \omega t}$

$$\rightarrow -\frac{n^2}{c^2}\omega^2 = \alpha = -k^2$$

$$\therefore \quad k = \pm \frac{n\omega}{c}$$

full solution $E_x = A e^{\pm i k z \pm i \omega t}$

<u>notes</u>

ω = angular frequency = $2\pi\nu$

= constant through diff't media

k = wave number = $\frac{2\pi}{\lambda} = n \cdot \frac{\omega}{c}$

define $k_0 \equiv \omega/c$ $\quad k = n k_0$ = wave number in medium

$\lambda = \lambda_0/n$ wavelength in medium is shorter

relative sign

$k z \mp \omega t \equiv \phi(z,t)$ = phase, position w/in 

↳ $-$ → wave moves in $+z$ direction

$+$ → wave moves in $-z$ direction.

We will use convention with $e^{+ikz - i\omega t}$

(Svelto uses $e^{-ikz + i\omega t}$)

c/n : phase velocity = v_{ph}

E-field is real, not complex
we use complex, phasor notation for convenience.
- take real part at the end.

can write $A e^{i(kz - \omega t + \phi_0)}$ then take real part.

or $A e^{i(kz - \omega t + \phi_0)} + c.c.$

can absorb phase into amplitude coeff.

$$A e^{i\phi_0} \rightarrow A \quad \text{(complex)}$$

$\hookrightarrow$ real $\hookrightarrow$ real

or can use cosines + sines:

$$A \cos(kz - \omega t + \phi_0)$$

$$\text{or} \quad A \cos(kz - \omega t) + B \sin(kz - \omega t)$$

There is also a wave eqn for B field, w/ same solution.
we will usually just work w/ E-field.

Resonator - 1 D, vacuum, perfectly conducting ends (mirro…

assume $E(0) = E(L_x) = 0$

- E-component tangent to conductor = 0

$$E_x(z,t) = \left(A_+ e^{+ik_z z} + A_- e^{-ik_z z}\right) e^{-i\omega t}$$

= superposition of +'ve, -'ve propagating waves.

For $E_x = 0$ at $z = 0$ $\rightarrow$ $A_+ = -A_-$ or write

$$E_x(z,t) = A \sin kz \, e^{-i\omega t}$$

For $E_x = 0$ at $z = L_x$ $\rightarrow$ $k_z L_x = \ell\pi \quad \ell = 1, 2, 3 \ldots$

or $k_z = \dfrac{\ell\pi}{L_x}$ or $\ell \dfrac{\lambda}{2} = L_x$ integer # half wavelengths.

if these are the allowed λ's what are the frequencies?

$$\frac{\ell\pi}{L_x} = \frac{\omega_\ell}{c}$$

these are the resonant frequencies

these are the longitudinal modes of the resonator.

equal spacing:

$$\Delta\omega = \frac{\pi c}{L_x} \quad \text{or} \quad \Delta\nu = \frac{c}{2L_x} = \frac{1}{\tau_{\text{round trip}}}$$