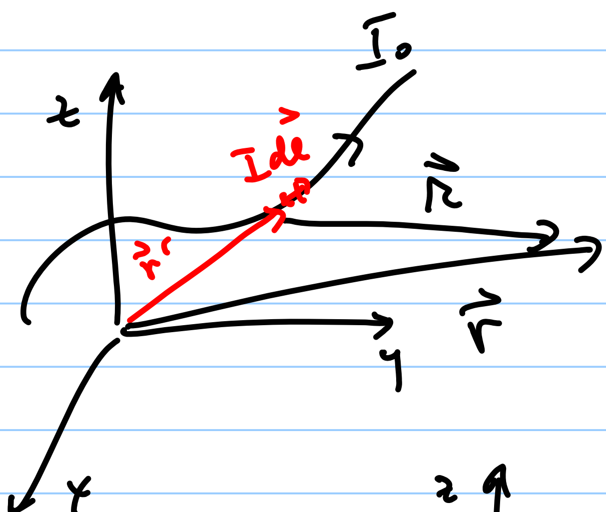
Overview

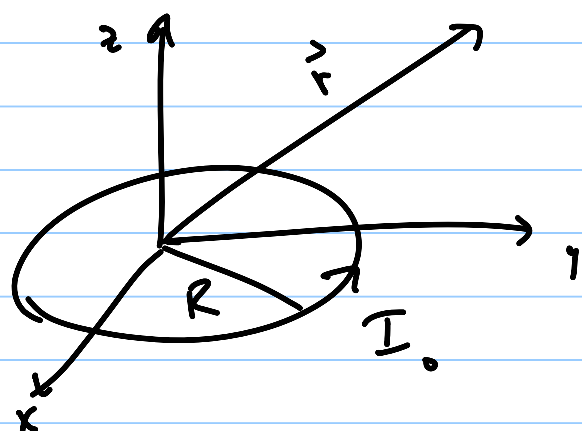
Note Title 3/19/2008

$$\vec{F} = q\vec{v}\times\vec{B} \xrightarrow[\text{wire}]{} \int \underbrace{I\,d\vec{l}'\times\vec{B}}_{\vec{I}\times\vec{B}\,dl'} \xrightarrow[\text{sheet}]{\text{current}} \int \vec{K}\times\vec{B}\,da' \longrightarrow \int \vec{J}\times\vec{B}\,d\tau'$$

Find $\vec{B}$ magnetostatics: $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\hat{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$

$$\longrightarrow \frac{\mu_0}{4\pi}\int \frac{\vec{K}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,da \longrightarrow \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

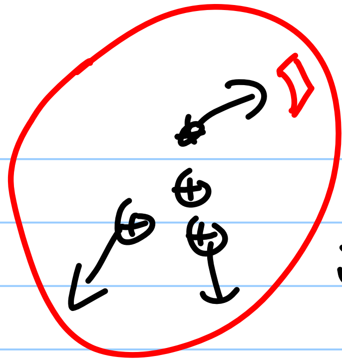




cons. of charge

$$\frac{e}{m^3}\,\frac{m}{s} \longrightarrow \frac{\text{amps}}{m^2}$$

$$\vec{J} = \underline{\rho v}$$



$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$ cons charge

integral form

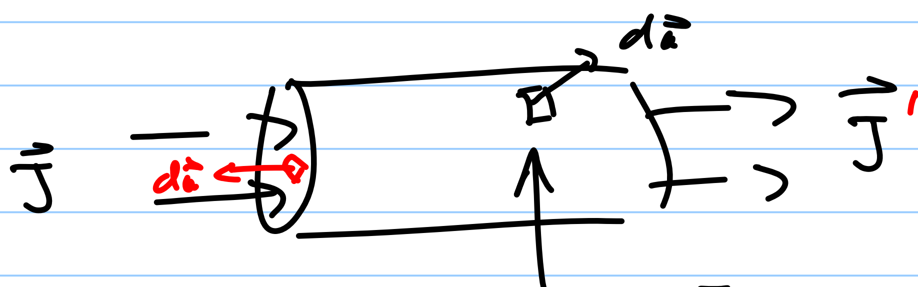
$$\underbrace{\int \vec{\nabla}\cdot\vec{J}\,d\tau}_{\oint \vec{J}\cdot d\vec{a}} = \underbrace{-\frac{\partial}{\partial t}\int \rho\, d\tau}_{Q_{encl.}}$$

$$\underbrace{\oint \vec{J}\cdot d\vec{a}}_{\text{flux of } J \text{ through tile}} = Q_{encl.}$$

Magnetostatics

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} \longrightarrow 0$$

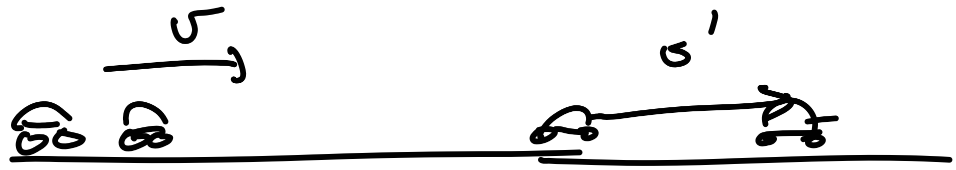
$$\int \vec{\nabla}\cdot\vec{J}\,d\tau = \oint \vec{J}\cdot d\vec{a} = 0$$



$$\oint \vec{J}\cdot d\vec{a} = \underbrace{\int_{cap} \vec{J}\cdot d\vec{a}}_{-J\pi r^2} + \cancel{\int_{body}}^{\,0} + \underbrace{\int_{cap}}_{J'\pi r^2}$$

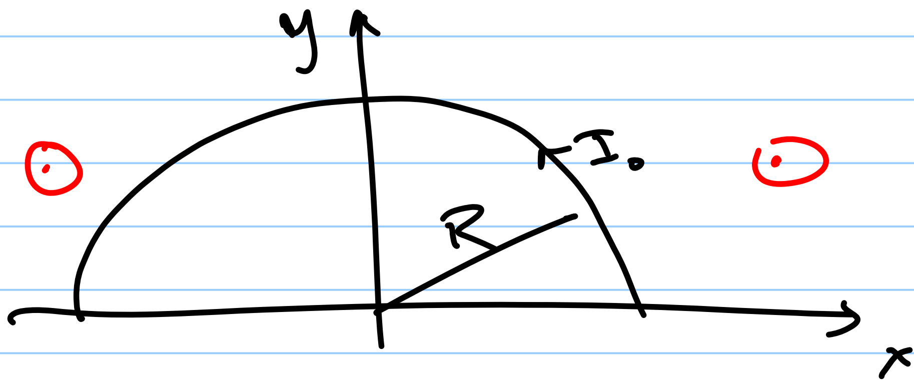
$$-J\pi r^2 + J'\pi r^2 = 0$$

$$\rho v = J = J' = \rho' v'$$



$\rho v \to \lambda v$ $\qquad \lambda > \lambda'$

$\lambda = \frac{\text{cars}}{\text{length}}$ $\qquad v < v'$



1.) Fundamental Principle

2.) method

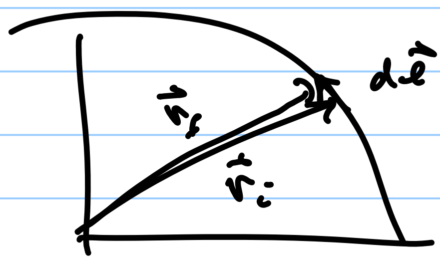
3.) how check answer

$\vec{B} = B_0 \hat{z}$

1.) $\vec{F} = \int I_0 \, d\vec{l} \times \vec{B}$

2.) find $d\vec{l}$ & limits take cross product

3.) $I_0 \to 0 \quad \vec{F} \to 0 \qquad R \to 0 ; \ \vec{F} \to 0$



$d\vec{l} = R \, d\theta \, \hat{\theta}$

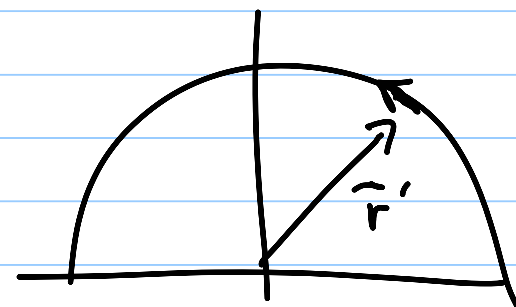
$$\boxed{\vec{r} = R\cos\theta \, \hat{x} + R\sin\theta \, \hat{y}}$$

$$d\vec{l} = d\vec{r} = R(-\sin\theta \, d\theta)\hat{x} + R\cos\theta \, d\theta \, \hat{y}$$

$$\vec{F} = \int I_0 \, d\vec{l} \times \vec{B} = \int_0^{\pi} I_0 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -R\sin\theta \, d\theta & R\cos\theta \, d\theta & \phi \\ 0 & 0 & B_0 \end{vmatrix}$$

Problem 2 find B at origin

1) $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$



$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{|\vec{\mathscr{r}}|}$

$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' = -\left(R\cos\theta \, \hat{x} + R\sin\theta \, \hat{y}\right)$

$\vec{r} = \phi$

Biot & Savart

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$\downarrow$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, d\tau'$$

$\boxed{\vec{\nabla} \cdot \vec{B} = \phi}$ $\qquad$ $\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}(\vec{r})}$

$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$