Nonlinear Optics
Homework 3
due Wednesday, 4 Feb 2009

- **Problem 1:**
  Boyd, problem 2.19

- **Problem 2:**
  In third-harmonic microscopy, harmonic light is generated when there is an interface in the focus. The signal is not really generated *at* the interface, but it is localized near there because of the role of phase matching. In this problem, we will consider the longitudinal spatial resolution of this kind of microscopy. Here are the parameters:
  - We focus a Gaussian beam with a vacuum wavelength of 1 micron at *f*/1 (the beam diameter to $1/e^2$ is equal to the focal length).
  - We are looking at a fused silica slide in water. Use the Sellmeier equations for fused silica shown below, and assume that in water n= 1.33 (no dispersion). You may assume that the thickness of the fused silica slide is much greater than the Rayleigh range of the focus. Be careful in how to introduce the refractive index of the slide into the equations for the focal spot size and the Rayleigh range: the wavelength that appears in these equations is its value in the medium i.e. $\lambda = \lambda_{vac}/n$.

  Calculate the third-harmonic signal vs. z-position as the focal spot is moved in the z-direction. The FWHM of this peak (full-width at half-maximum) is effectively the resolution of the imaging system in the longitudinal direction. We are not interested in the actual strength of the signal, just how it varies with z. Use the absolute value of the expression 2.10.11b.

- **dispersion for fused silica**

```
as1 = .6961663;
as2 = .4079426;
as3 = .8974794;
bs1 = .0684043^2;
bs2 = .1162414^2;
bs3 = 9.896161^2;
```

$$ns[\lambda\_] := \sqrt{\left(1 + \frac{as1\,\lambda^2}{\lambda^2 - bs1} + \frac{as2\,\lambda^2}{\lambda^2 - bs2} + \frac{as3\,\lambda^2}{\lambda^2 - bs3}\right)};$$

- **Problem 3:**
  Consider wave mixing in a square cross-section gas-filled hollow waveguide.

  Assuming the field goes to zero at the boundaries of the waveguide, the propagation constant and the eigenmodes for a gas-filled waveguide are calculated as follows:

From the vector relationship for the wavevector components:
$kz^2 = k0^2\, n^2 - kx^2 - ky^2$

where n is the refractive index of the gas:
$n^2 = 1 + p\,\mathrm{dn}$ **, where dn > 0**
and
$kx = mx\, \pi / L$
$ky = my\, \pi / L$
This results in:
$kz = \beta = \sqrt{k0^2 (1 + p\,\mathrm{dn}) - (mx\, \pi / L)^2 - (my\, \pi / L)^2}$

Note that the deviation from the vacuum wavenumber is very small. We can expand the square root out:
$\beta = k0 \sqrt{1 + p\,\mathrm{dn} - (mx\, \pi / k0\, L)^2 - (my\, \pi / k0\, L)^2} = k0\left(1 + p\,\mathrm{dn}/2 - \frac{1}{2}\left(\frac{\pi}{k0\, L}\right)^2 \left(mx^2 + my^2\right)\right)$

a. Using this information, derive a phase matching condition for third-harmonic generation. Assuming the fundamental is in a single, well-defined mode, to what modes can the phase matching be satisfied for the 3rd harmonic? (Note that the pressure is variable, p > 0).

b. Assuming the input beam is in the lowest-order mode, calculate an expression for the mode overlap factor for the modes that can be phasematched. Calculate the ratio between the overlap integral for the best output mode, and compare it with the next best one.

c. Now consider a 4-wave mixing experiment, where the second harmonic is mixed with the fundamental to produce the 3rd harmonic: $\omega_3 = 2\,\omega_2 - \omega_1$**. You should see that it is possible to achieve phase matching if all beams are in the lowest order spatial mode. Calculate the mode overlap factor in this case.**

It is not necessary to evaluate the phase-matching pressure, but if you are interested, an expression for the dispersion of argon is below.

- **Argon dispersion equations (good down to 110 nm)**
  wavelength in microns

$$\mathrm{dnAr}[\lambda\_] := 2 * 120.625\ 10^{-6} \left(\frac{14.0}{87.892 - (\lambda)^{-2}} + \frac{14.0}{91.011 - (\lambda)^{-2}} + \frac{430.63}{217.6 - (\lambda)^{-2}}\right);$$

```
nAr[λ_, pr_] = 1 + pr dnAr[λ];
```