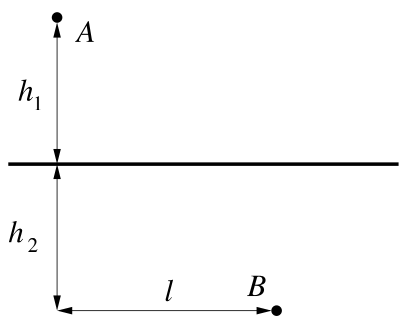
Problem Set 3

Due: Tuesday, July 10th, at the *beginning* of class.

1. Chapter 9: 22, 23, 27, 28, 30, 31 (b), 37
2. Use the Schrodinger dispersion relation for two (or more) harmonic waves with slightly different frequency to make a wavegroup. Use Mathematica to animate the motion of the wavegroup. Does a wavecrest move through the wavegroup, or the wavegroup move through the wavecrest, or do both move at the same speed? Submit your code with the output deleted.
3. **Fermat's Principle of Least Time**
   Fermat's principle or the principle of least time is the idea that the path taken between two points by a ray of light is the path that can be traversed in the least time. Imagine you have a light ray that starts at point A in medium 1 (with speed v_1) and is to travel to point B in medium 2 (with speed v_2). Use Fermat's principle to show that the light will follow the path given by Snell's law.



4. **Working with the Jones Vectors**
   The electric vector of a wave is given by the real expression

$$\vec{E} = E_0 \left[\cos\left(kz - \omega t\right)\hat{x} + b\cos\left(kz - \omega t + \delta\right)\hat{y}\right].$$

   (a) Sketch diagrams to show the type of polarization and give the Jones vectors for the following cases
      i. $\delta = 0, b = 1$
      ii. $\delta = 0, b = 2$
      iii. $\delta = \pi/2, b = -1$
      iv. $\delta = \pi/4, b = 1$

   (b) Describe the type of polarizations of the waves with the following Jones vectors. Also find the orthogonal Jones vectors to the ones shown and describe their polarizations.

$$\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, \begin{pmatrix} i \\ -1 \end{pmatrix}, \begin{pmatrix} 1-i \\ 1+i \end{pmatrix}$$

5. **Jones Matricies**
   (a) Show that circularly polarized light is produced by sending light through a linear polarizer and a quarter-wave plate *only* in the right order.

   (b) Linearly polarized light whose Jones vector is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (horizontally polarized) is sent through a train of two linear polarizers. The first is oriented with its transmission axis at 45 degrees and the second has its transmission axis vertical. Show that the emerging light is linearly polarized in the vertical direction. Discuss the intensity change through each polarizer.

6. **Eigenvectors of Jones Matricies**

   (a) Find the eigenvalues and corresponding eigenvectors for a left circular polarizer. Discuss the physical interpretation of these eigenvalues and eigenvectors.

   (b) Find the eigenvalues and corresponding eigenvectors for a linear polarizer with its transmission axis at 45 degrees. Discuss the physical interpretation of these eigenvalues and eigenvectors

Extra Credit (Courtesy of Dr. Wood)

Some err over the rainbow

The primary rainbow is the result of two refractions and an internal reflection inside (we assume, spherical) raindrops. (Like people, only when they are quite large do they begin to slump due to gravity.) White light from the Sun is incident as on the surface of a raindrop at an angle θ_{incid} as shown; it is transmitted at an angle θ_{trans} (which actually depends on the wavelength of the light). The overall geometry is shown in the first figure at the end of the question.

(a) Examine the second figure at the end of the question. Show that the primary rainbow angle θ_{prim} obeys

$$\frac{1}{2}\theta_{\text{prim}} = 2\theta_{\text{trans}} - \theta_{\text{incid}}.$$

[*Hint:* the situation is symmetrical about the point of internal reflection, as indicated, so you can focus on the half-angle.] Since we *know* θ_{trans} from Snell's Law, we know $\frac{1}{2}\theta_{\text{prim}}$ as a function of θ_{incid}.

(b) Using an index of refraction $n \simeq \frac{4}{3}$ for water and 1 for air, plot $\theta_{\text{prim}}(\theta_{\text{incid}})$ as θ_{incid} ranges from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$ and show that θ_{prim} has a maximum as a function of θ_{incid}. Since it is (or should be!) a fairly broad maximum, a large range of incident rays are 'collected' to this value of θ_{prim}, making the deflected light at this angle most intense.

(c) What happens when the sun makes an angle greater than this angle?

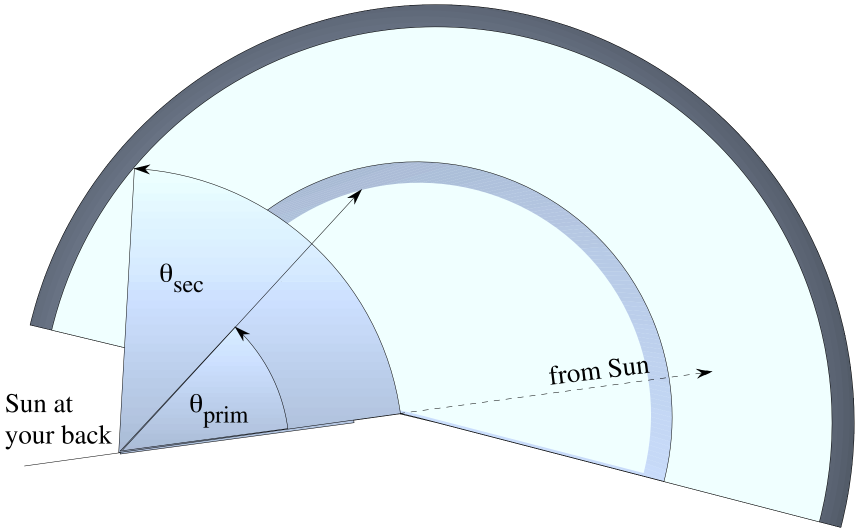
(d) Find an expression for the maximum value of θ_{prim} valid for general index of refraction n of the liquid making up the droplet. For water, find this angle in degrees. Compare this value with Brewster's angle for the water/air interface. What do you think this will imply about the light making up the rainbow?

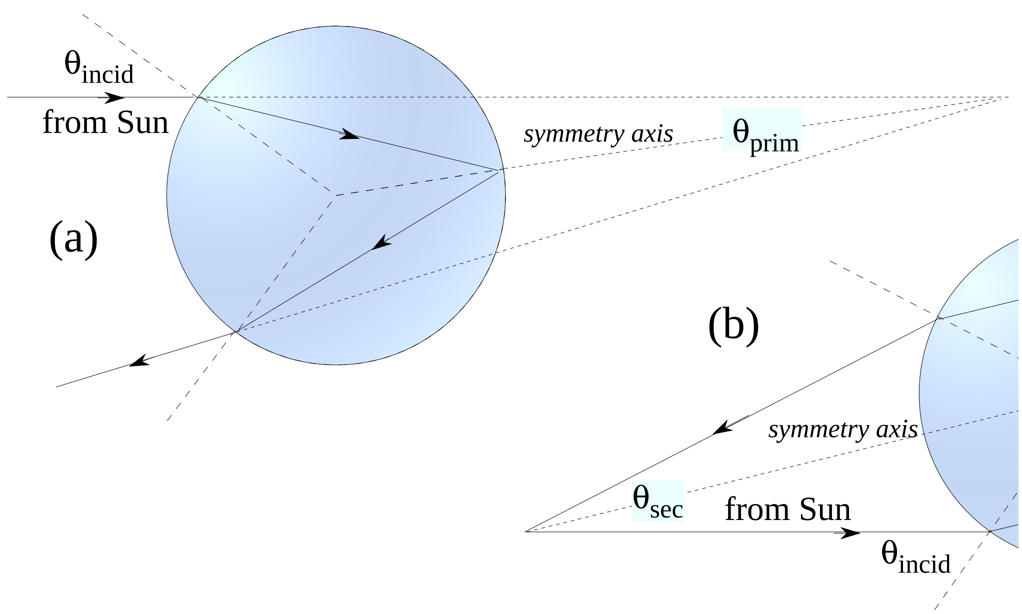
(e) The index of refraction of water actually depends on the wavelength of light, ranging from about 1.344 for violet light to about 1.331 for red light. So light of which color do you expect to be lower in the sky?

(f) Sorry about the figure being cutoff for this one. I put a little picture to get across what you have missed. Use the same sort of analysis to show that there is a *secondary* rainbow at an angle given by

$$\theta_{\text{second}} = \pi + 2\left(\theta_{\text{incid}}. - 3\theta_{\text{trans}}\right),$$

Why should this be much fainter than the primary rainbow, with a dispersion in colors opposite that for the primary rainbow? [In fact, it should be separated from the primary rainbow by a band which you can predict should be dark ('Alexander's dark (ragtime) band')]


θsec
θprim
Sun at
your back
from Sun


θincid
from Sun
symmetry axis
θprim
(a)
(b)
symmetry axis
θsec
from Sun
θincid