MATH 225 - Differential Equations May 13, 2008

Homework 2, Field 2008 **Not Due Date**: May 14, 2008

MATHEMATICAL MODELS - SEPARATION OF VARIABLES - SLOPE FIELDS - EULER'S METHOD

1. Solutions to ordinary differential equations.

   (a) Show that $y(t) = e^{2t} + ce^t$, where $c \in \mathbb{R}$, is a solution to $\dfrac{dy}{dt} - y = e^{2t}$.

   (b) Show that $x^2 + y^2 = cx$, where $c \in \mathbb{R}$, is a solution to $2xy\dfrac{dy}{dx} = y^2 - x^2$.

   (c) Show that $y(t) = c_1\text{sinh}(t) + c_2\text{cosh}(t)$, where $c_1, c_2 \in \mathbb{R}$, is a solution to $y'' - y = 0$.

   (d) Show that $y(t) = c_1 \sin(t) + c_2 \cos(t)$, where $c_1, c_2 \in \mathbb{R}$, is a solution to $y'' + y = 0$.

   Hint: For problem 1c recall that $\sinh(t) = \dfrac{e^t - e^{-t}}{2}$, $\cosh(t) = \dfrac{e^t + e^{-t}}{2}$.

2. The fox squirrel is a small mammal native to the Rocky Mountains. These squirrels are very territorial, so that if their population is large, their rate of growth decreases and may even become negative. On the other and , if the population is too small, fertile adults run the risk of not being able to find suitable mates, so again the rate of growth is negative.

   (a) The fox squirrel population dynamics can be modeled with a modified logistics equation. List your assumptions, variables/parameters and define a first-order ODE governing the dynamics of the fox squirrel population.

   (b) On a population-time plane sketch the possible solutions to the ODE. Be sure to include any equilibrium solutions and solutions representative of qualitatively different regions.

   (c) Using separation of variables solve the ODE created in part (a).

   Hint: Do not find explicit solutions to the result from(c), unless you really want to,

3. The following nonlinear system has been proposed as a model for a predator-prey system of two particular species of microorganisms.

$$\frac{dx}{dt} = ax - by\sqrt{x} \qquad (1)$$

$$\frac{dy}{dt} = cy\sqrt{x}, \qquad (2)$$

   where $a, b, c \in \mathbb{R}^+$. In this case the variables x and y are dependent variables and appear in both ODE's and thus the ODE's are said to be coupled.

   (a) Which variable, x or y, represents the predator population? Which variable represents the prey population? Justify your choices.

   (b) What happens to the predator population if the prey is extinct? Justify your conclusion.

4. Solve the following problems via separation of variables. When appropriate solve for the integrating constant C using the initial value which is given.

   (a) $\frac{dy}{dt} = 1 + \frac{1}{y^2}$.

   (b) $(y')^2 - xy' + y = 0$.

   (c) $\frac{dy}{dt} = (y^2+1)t, \;\; y(0) = 1$.

   (d) $\frac{dy}{dt} = \frac{ye^t}{1+y^2}$.

   Hint: For (b) consider completing the square and using the variable substitution $z = -(y - t^2/4)$.

5. Consider the polynomial $p(y) = -y^3 - 2y + 2$.

   (a) Using HPGSolver sketch the slope field for $\frac{dy}{dt} = p(y)$.

   (b) Using HPGSolver, sketch the graphs of some of the solutions using the slope field.

   (c) Describe the relationship between the roots of $p(y)$ and the solutions of the differential equation.

   (d) Using Euler's method, approximate the real root(s) of $p(y)$ to three decimal places.