MATH 225 - June 5, 2008 **NAME**:
Exam III - 50 minutes - 50 Points

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) Solve the following second-order ordinary differential equations.
   (a) $y'' + y = 0$

   (b) $y'' - y = 0$

   (c) $y'' = 0$

2. (10 Points) Given that $y'' + 4y' + 4y = f(t)$.
   (a) Find the homogeneous solution to the ODE.

   (b) Write down the form of the particular solution supposing that $f(t)$ is given by:
      i. $f(t) = 2e^{-t}$
      ii. $f(t) = 3$
      iii. $f(t) = 5e^{-2t}$
      iv. $f(t) = 3\cos(3t)$

   DO NOT SOLVE FOR THE UNKNOWN CONSTANTS. IF USING IMAGINARY EXPONENTIALS BE SURE TO INCLUDE WHETHER THE REAL OR IMAGINARY PART SHOULD BE KEPT.

3. (10 Points) Solve the following initial value problem.

$$2y'' - 8y = 16 - 18e^{-t}, \;\; y(0) = 1, \;\; y'(0) = -3 \qquad (1)$$

4. (10 Points) Given,

$$y' + 2y = 0. \qquad (2)$$

(a) Assume a power-series solution to the ODE and find the corresponding recurrence relation for the power-series coefficients.

(b) Solve the recurrence relation for these coefficients and using a known Taylor series find a transcendental expression for your solution.

(c) Check your result.

5. Given the following forced simple harmonic oscillator.

$$2\frac{d^2y}{dt^2} + 8y = 6\cos(\omega t), \quad y(0) = 0, \;\; y'(0) = 0. \tag{3}$$

(a) Set $\omega = 1$ and find the solution to the initial value problem.

(b) Set $\omega = 2$ and find the solution to the initial value problem.

(c) Describe the qualitative differences between these solutions.

6. (10 Points - Extra Credit) Given,

$$y'' + y = 0, \quad y(0) = -1,\ y'(0) = 1. \tag{4}$$

(a) Convert the second-order ODE into a system of first order ODE's.

(b) Using eigenvalues and eigenvectors, solve the corresponding initial value problem. Express you solution in real form.