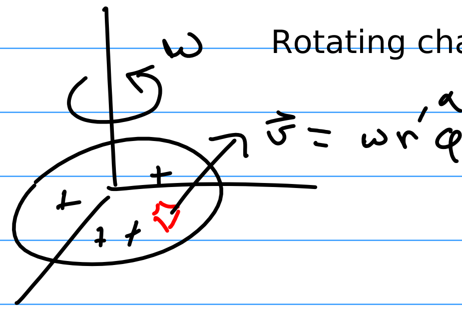
Lecture 12 Review and continue with Shadowitz: Chapter 4-1

Exam schedule: Feb. 7 exam 2 and March 7 exam 3.

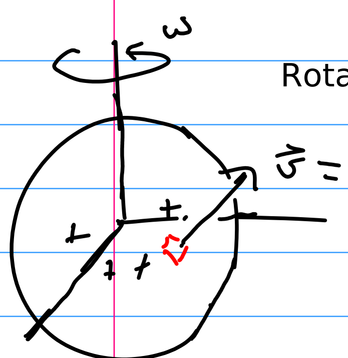
Formulas: Memorize Stokes and the divergence theorems. Also memorize Gauss's and Ampere's laws along with Biot-Savart for line currents. Memorize forces on surface and volume currents along with the Lorentz force. All calculations involving the divergence and curl will be in cartesian coordinates. However, you should be prepared to apply Ampere's and Gauss's laws in other coordinate systems.

Check out new supplementary material about vector calculus on the wiki.

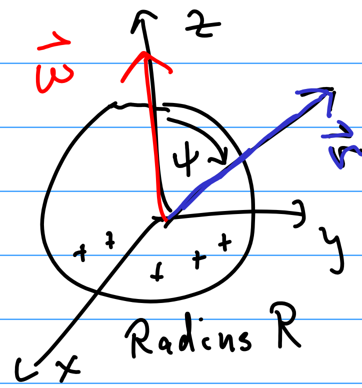
Homework problem:

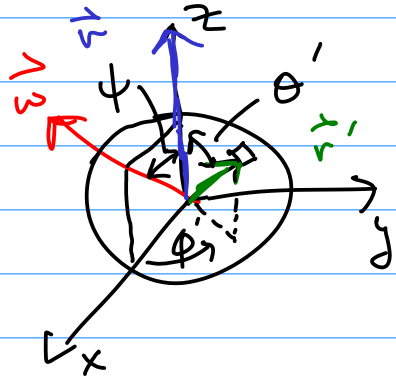


Rotating charged record



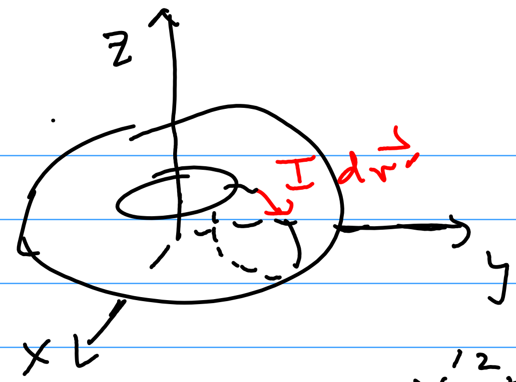
Rotating charged spherical shell



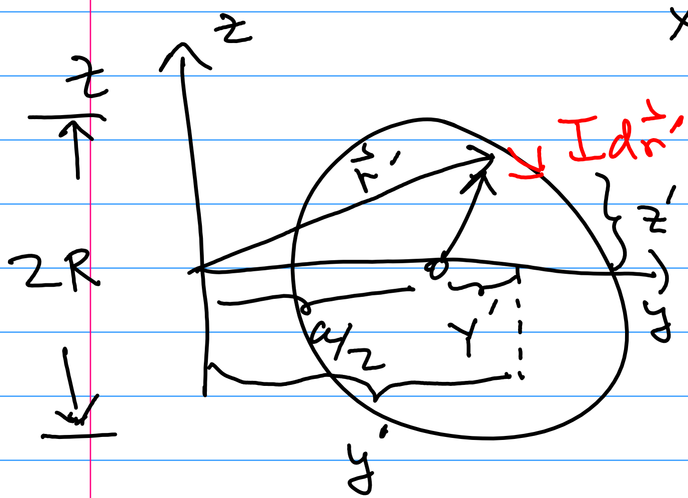


Let $\vec{\omega}$ be in the x-z

Homework problem:



$$Y'^2 + z'^2 = R^2$$



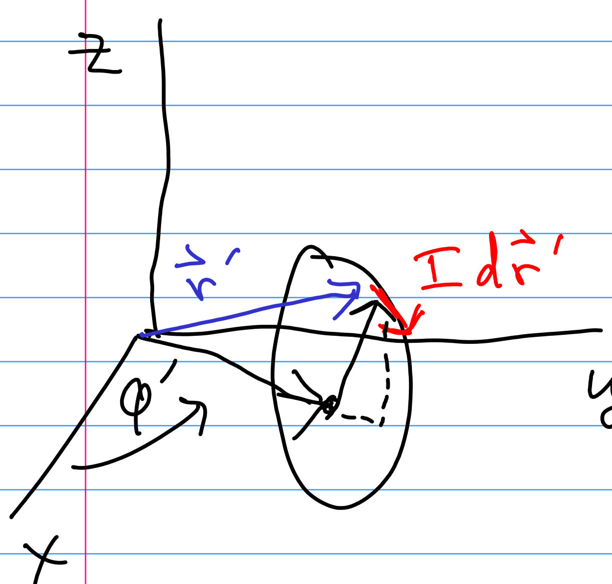
$$\vec{r}' = y'\hat{y} + z'\hat{z}$$

$$\vec{r}' = \left(\frac{a}{2} + Y'\right)\hat{y} + \sqrt{R^2 - Y'^2}\,\hat{z}$$

$$d\vec{r}' = dY'\hat{y} + \frac{1}{2}\frac{-2Y'\,dY'}{\sqrt{}}\hat{z}$$

This looks like a multivalued function. However sign is needed for the lower half of circle.

$$\vec{r}' = \left(\frac{a}{2} + Y'\right)\hat{y} \pm \sqrt{R^2 - Y'^2}\,\hat{z}$$



$$\vec{r}' = \left(\frac{a}{2} + Y'\right)\cos\varphi'\,\hat{x} + \left(\frac{a}{2} + Y'\right)\sin\varphi'\,\hat{y} + \sqrt{R^2 - Y'^2}\,\hat{z}$$

$$df = \frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y$$

$\vec{r}'$

Statement $\vec{\nabla} \times \vec{\nabla} V(x,y,z) = 0$

Questions: How do you calculate or show this (congruous)? Just write it out in cartesian coords.

$$\vec{\nabla} V = \hat{x}\frac{\partial}{\partial x}V + \hat{y}\frac{\partial}{\partial y}V + \hat{z}\frac{\partial}{\partial z}V$$

$$\vec{\nabla} \times \vec{\nabla} V(x,y,z) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \end{vmatrix}$$

$$= \hat{x}\underbrace{\left(\frac{\partial^2 V}{\partial z \partial y} - \frac{\partial^2 V}{\partial y \partial z}\right)}_{\phi} - \hat{y}\underbrace{\left(\frac{\partial^2 V}{\partial x \partial z} - \frac{\partial^2 V}{\partial z \partial x}\right)}_{0} + \underbrace{\hat{z}(\quad)}_{0}$$

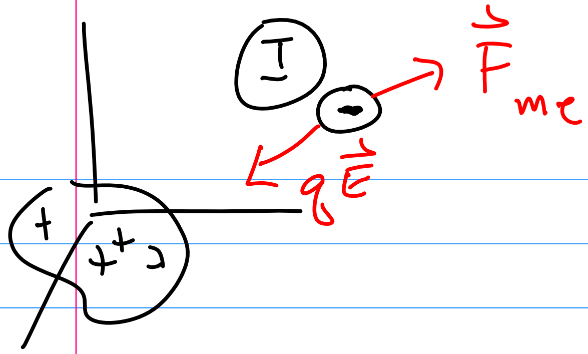
$$\vec{\nabla} \times \vec{E} = 0$$

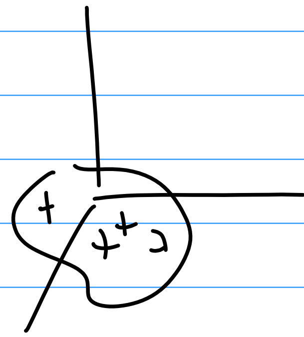
We can therefore write $\vec{E} = -\vec{\nabla} V$

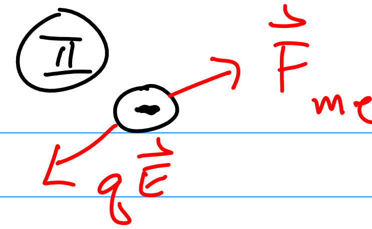
$\uparrow$ convention

$$\int \vec{\nabla} \times \vec{E} \cdot d\vec{a} = \oint \vec{E} \cdot d\vec{r}$$

$$-\int \underbrace{\vec{\nabla} \times \vec{\nabla} V}_{\phi} \cdot d\vec{a} = \oint \vec{E} \cdot d\vec{r} = 0$$





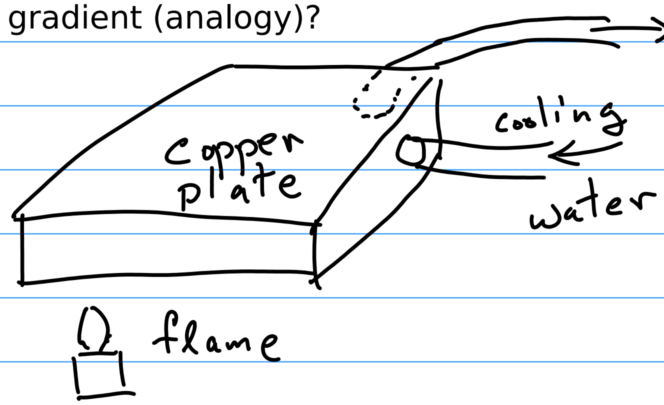


$$\int_{I}^{II} \vec{F}_{me} \cdot d\vec{r} = \text{work done by me in moving charge at constant speed from I to II}$$

$$= q\left(-\int_{I}^{II} \vec{E} \cdot dr\right) = q \int_{I}^{II} \vec{\nabla} V \cdot d\vec{r}$$

What analogy gives physical intuition about the gradient (analogy)?

Use temperature T rather than voltage V.

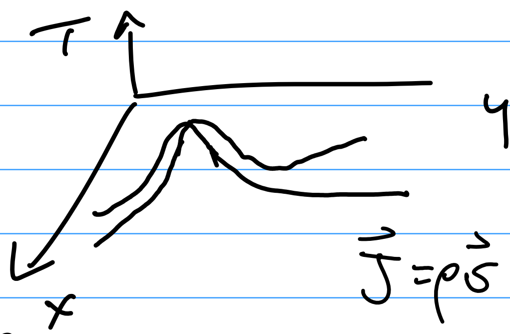


scalar function $T(x,y)$

For many materials we can write

$$\vec{h} = k \vec{\nabla} T(x,y) \quad \frac{\text{Joules}}{s \cdot m^2}$$

Thermal conductivity



$$\vec{J} = \rho \vec{v} \quad \frac{C}{m^3} \frac{m}{s}$$

Called heat flux but what is it (informational)?

$$\int \vec{h} \cdot d\vec{a}$$

$$\int \vec{J} \cdot d\vec{a} \text{ flux}$$

What equation represents heat (energy) flux (informational)?

If heat energy is conserved. That is no sources (flames) or sinks (cooling water).

thermal energy within surface

thermal energy per volume (density)

$$\oint \vec{h} \cdot d\vec{a} = -\frac{dG}{dt} = -\frac{d}{dt} \int g \, dVolume$$

If there is a source or sink

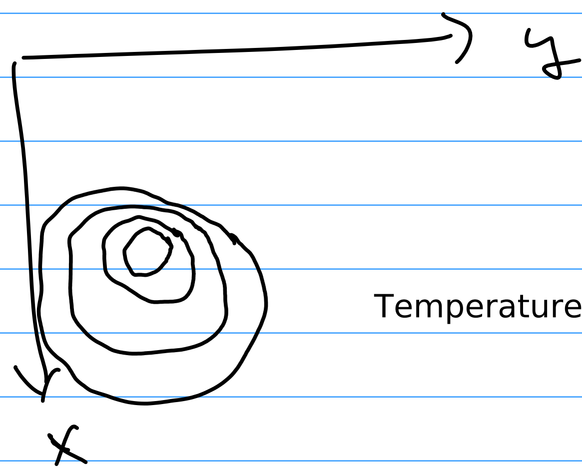
$$\oint \vec{h} \cdot d\vec{a} = -\frac{dG}{dt} + S$$

What's the differential form using the divergence theorem (congruous)?

$$\int \vec{\nabla} \cdot \vec{h} \, dvol = -\frac{d}{dt} \int g \, dvol \Rightarrow \vec{\nabla} \cdot \vec{h} = -\frac{\partial}{\partial t} g$$

Conservation of charge analogy is

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial}{\partial t} \rho$$



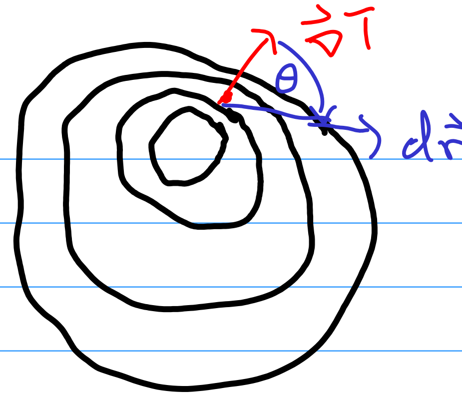
Temperature contour plot

$$\vec{\nabla} T \cdot d\vec{r} = \left(\hat{x} \frac{\partial T}{\partial x} + \hat{y} \frac{\partial T}{\partial y} + \hat{z} \frac{\partial T}{\partial z} \right) \cdot \left(dx\,\hat{x} + dy\,\hat{y} + d\,\hat{z} \right)$$

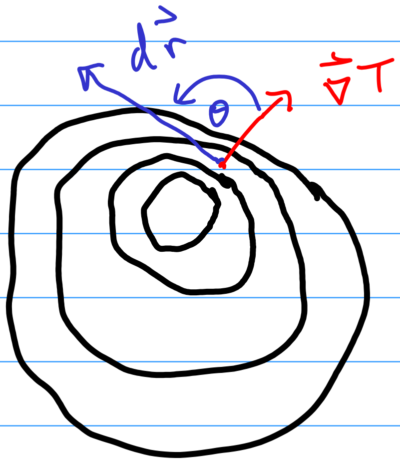
$$= \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz = dT$$

dT

$$dT = \vec{\nabla} T \cdot d\vec{r} = |\vec{\nabla} T|\,|d\vec{r}| \cos\theta$$



Fix the magnitude of dr and change theta.



For what direction of dr is the change in T the greatest (informational)?

dT is greatest when dr moves in direction of grad T

Fundamental theorem of gradients.

$$\vec{\nabla} T \cdot d\vec{r} = \left(\hat{x}\frac{\partial T}{\partial x} + \hat{y}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z}\right) \cdot \left(dx\,\hat{x} + dy\,\hat{y} + d\hat{z}\right)$$

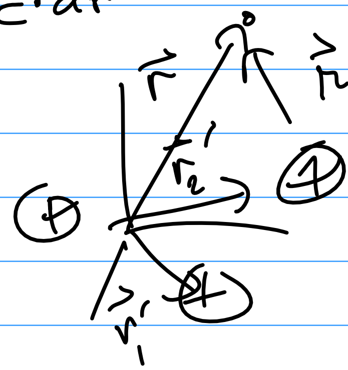
$$= \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz = dT$$

Differential form $\quad \vec{E} = -\vec{\nabla} V$

Integral form $\quad \int \vec{E} \cdot d\vec{r} = -\int \underbrace{\vec{\nabla} V \cdot d\vec{r}}_{dV} = -(V_f - V_i)$

pt charge: $$V(r) = -\int_{\infty}^{r_f} \underbrace{\frac{kq}{r^2}\,dr}_{\vec{E}\cdot d\vec{r}} = k\frac{q}{r_f}$$

$$V(r=\infty) = 0$$



Superpositon

$$V(r) = \sum_i \frac{kq_i}{r_i} \longrightarrow \int \frac{k}{r}\,dq$$