Homework 6 — due 5 October 2007 by 5pm
PH462 EM Waves and Optical Physics — posted: 29 September 2007

Reading: Heald and Marion (HM) chapter 7 and posted notes.

1) Consider a rectangular waveguide for a scalar field (as we discussed initially in class), where the field goes to zero at the boundaries. The width and height of the waveguide are a and b, respectively, and the medium within the waveguide has a refractive index n.
    a. Write down or calculate a complex expression for the guided wave, defining all Cartesian components of k.
    b. Calculate expressions for the phase and group velocities, and compare each of them to their values in a bulk medium of refractive index n.

2) HM 7-9
3) HM 7-10. See eqn 1.95 for how to calculate the surface currents.
4) HM 7-13
5) HM 7-14
6) Calculate the conditions on the TM bound modes for a planar dielectric waveguide. The **cladding and core indices are n_1 and n_2,** respectively, and the walls of the core are at $x = \pm a$. Derive the conditions on the allowed *symmetric* modes:
$$\frac{\alpha\, a}{n_2^2}\tan\alpha\, a = \frac{\beta\, a}{n_1^2}$$
Inside the waveguide core, use cos(αx) for the field; outside in the cladding, use exp[-β|x|]. In this calculation, solve for the B field, since in this case $\mathbf{B} = \hat{\mathbf{y}}B_y$ only. To get the condition on the derivative of B_y across the interface, use Maxwell's equation for the curl of **B**.