PHGN341 Thermal Physics Exam 1

(maximum = 60 points)

1.

Consider an N particle system. Each particle can be in one of four states. Three states have energy J and one state has energy $-3J$.

(a) Find the canonical partition function, Z, in terms of J, N, and temperature τ. (4 points)

(b) Find the internal energy U. (4 points)

(c) Find the Helmholtz free energy F. (4 points)

(d) Find the entropy, σ. (4 points)

(e) Determine the behavior of U and σ as $\tau \to 0$. (4 points)

2.

An ideal gas of Argon atoms is in equilibrium with argon atoms adsorbed on a planar surface. The argon gas has density n_g. There are N_s sites at which the argon atoms can be adsorbed. When the atoms are adsorbed, they each have energy $-I$.

(a) What is the chemical potential for the argon gas in terms of temperature τ, n_g, and the quantum density n_Q? (4 points)

(b) What is the chemical potential for the adsorbed argon atoms? (4 points)

(c) What is the probability that an argon atom is adsorbed? (4 points)

(d) What is the number N_{ad} of adsorbed argon atoms? (4 points)

(e) What is the classical limit for N_{ad}? (4 points)

3.

The thermodynamic identity for a one dimensional system is

$$\tau d\sigma = dU - f d\ell$$

where f is the external force exerted on the line and $d\ell$ is the extension of the line. The entropy $\sigma = \sigma(\ell, U)$.

(a) Show that

$$\left(\frac{\partial \sigma}{\partial \ell}\right)_U = -\frac{f}{\tau}$$

(5 points)

(b) Consider a polymer chain (such as composes rubber, for example) of N links (take N even), each of length a. Each link is equally likely to be directed to the left or to the right. Show that the number of arrangements that give a head-to-tail length $\ell = 2|s|a$ is

$$g(N,s) + g(N,-s) = 2g(N,|s|) = 2\frac{N!}{(\frac{1}{2}N + s)!(\frac{1}{2}N - s)!}$$

(Hint: This is the independent spin problem. Here, two opposite spins represent two missing links in the chain, so that links appear and disappear two at a time) (5 points)

(c) Find the equation of state for the chain (i.e., f as a function of ℓ, $L = Na$, and τ). Use

$$\sigma(\ell, U) = \ln(2g(N,|s|)) = (N+1)\ln(2) - \frac{N}{2}\left[(1+\frac{\ell}{L})\ln(1+\frac{\ell}{L}) + (1-\frac{\ell}{L})\ln(1-\frac{\ell}{L})\right]$$

(5 points)

(d) Find ℓ as a function of f, τ, and L. If this is a good model of a rubber band, what happens to the rubber band as temperature increases? (5 points)