

Graphical approach to convolution

Input functions

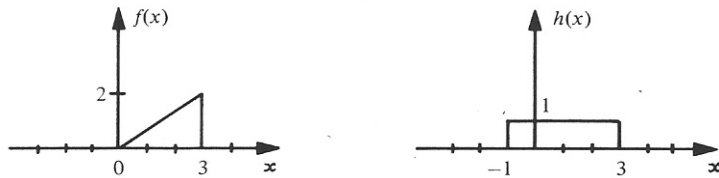


Figure 6-1 Functions used to illustrate convolution by graphical methods.

output

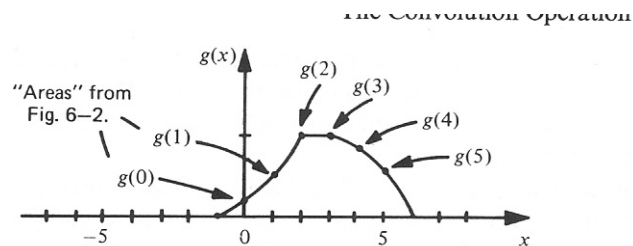


Figure 6-3 Resulting convolution of functions shown in Fig. 6-1.

Graphical view

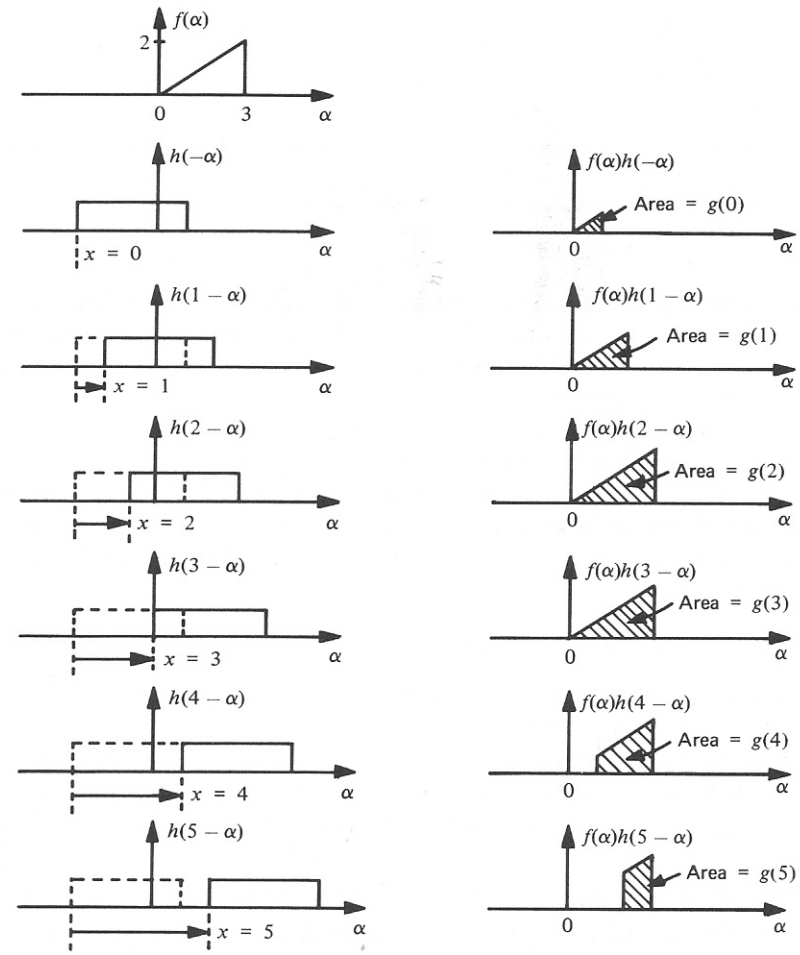


Figure 6-2 Graphical method for convolving functions of Fig. 6-1.

Smoothing effects

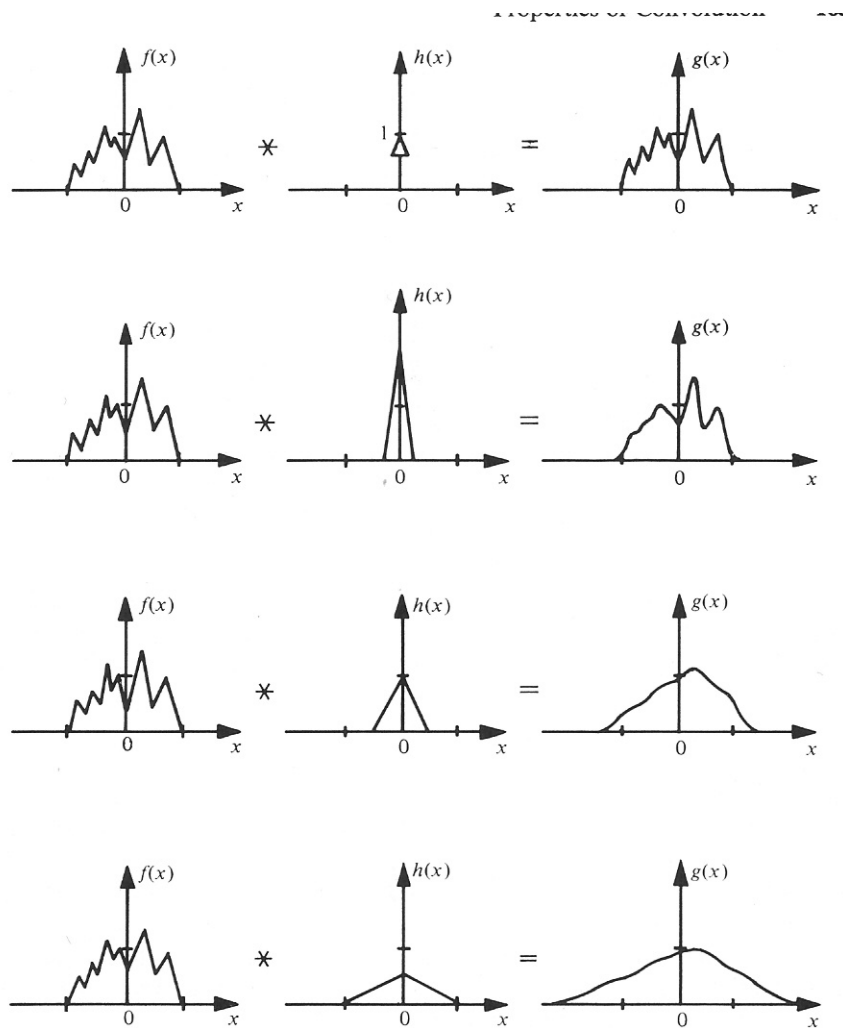


Figure 6-5 Smoothing effects of convolution.

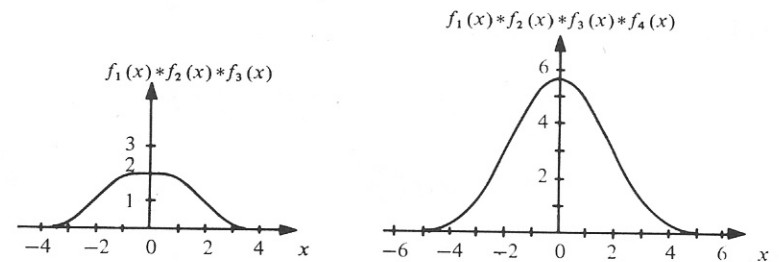
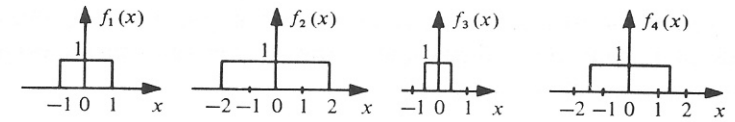


Figure 6-7 Repeated convolution of four rectangle functions.

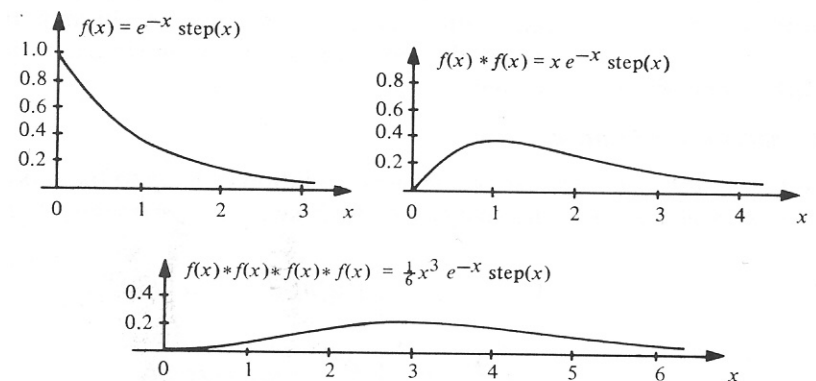


Figure 6-8 Repeated convolution of the function $\exp\{-x\}\text{step}(x)$.

Impulse response of an LSI system

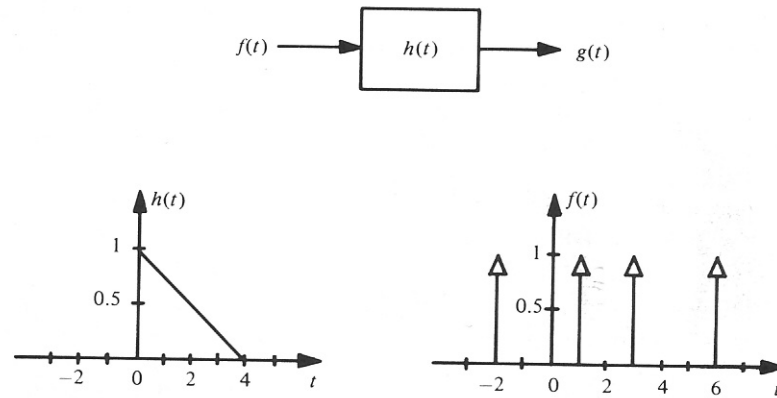


Figure 6-9 An example involving a linear shift invariant system.

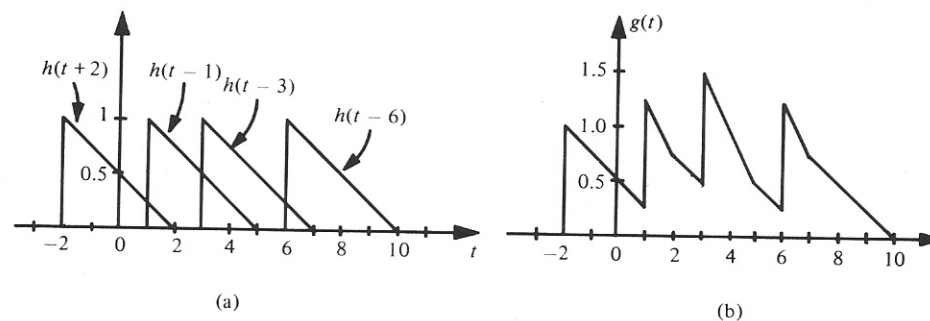


Figure 6-10 Output of system shown in Fig. 6-9. (a) Responses to individual input delta functions. (b) Overall response to combined input delta functions.

Symmetry properties of FT

Symmetry Properties of Fourier Transforms

| $f(x)$ | $F(\xi)$ |
|------------------------|------------------------|
| Complex, no symmetry | Complex, no symmetry |
| Hermitian | Real, no symmetry |
| Antihermitian | Imaginary, no symmetry |
| Complex, even | Complex, even |
| Complex, odd | Complex, odd |
| Real, no symmetry | Hermitian |
| Real, even | Real, even |
| Real, odd | Imaginary, odd |
| Imaginary, no symmetry | Antihermitian |
| Imaginary, even | Imaginary, even |
| Imaginary, odd | Real, odd |

FT theorems

Properties of Fourier Transforms

A_1 and A_2 arbitrary constants
 b and d real nonzero constants

x_0 and ξ_0 real constants
 k a positive integer

| $g(x) = \int_{-\infty}^{\infty} G(\beta) e^{j2\pi\beta x} d\beta$ | $G(\xi) = \int_{-\infty}^{\infty} g(\alpha) e^{-j2\pi\alpha\xi} d\alpha$ | |
|---|--|---------|
| $f(\pm x)$ | $F(\pm \xi)$ | |
| $f^*(\pm x)$ | $F^*(\mp \xi)$ | |
| $F(\pm x)$ | $f(\mp \xi)$ | |
| $F^*(\pm x)$ | $f^*(\pm \xi)$ | |
| $f\left(\frac{x}{b}\right)$ | $ b F(b\xi)$ | scaling |
| $ d f(dx)$ | $F\left(\frac{\xi}{d}\right)$ | |
| $f(x \pm x_0)$ | $e^{\pm j2\pi x_0 \xi} F(\xi)$ | shift |
| $e^{\pm j2\pi \xi_0 x} f(x)$ | $F(\xi \mp \xi_0)$ | |