Solution of coupled mixing eqns – no phase mismatch

3-wave mixing, SVEA: $\omega_3 = \omega_1 + \omega_2$

let $\xi = 8\pi d_{eff}/c = \boxed{\frac{2 d_{eff}}{c} \text{ SI}}$ $\quad \omega_3 > \omega_1, \omega_2$

$$A_1' = i \xi \frac{\omega_1}{n_1} A_3 A_2^* e^{-i\Delta k z}$$ $\;3 \uparrow \downarrow 2, \downarrow 1$ valid for any input.

SFG: $\omega_1 + \omega_2 \to \omega_3$

$$A_2' = i \xi \frac{\omega_2}{n_2} A_3 A_1^* e^{-i\Delta k z}$$ $\;3 \uparrow \downarrow 1, \downarrow 2$

DFG: $\omega_3 - \omega_2 \to \omega_1$

$\Delta k = k_1 + k_2 - k_3$

$$A_3' = i \xi \frac{\omega_3}{n_3} A_1 A_2 e^{i\Delta k z}$$ $\;2 \uparrow, 1 \uparrow \downarrow 3$

SHG: A_1, A_2 only $\quad 2\omega_1 \to \omega_2$

$$A_1' = i \xi \frac{\omega_1}{n_1} A_2 A_1^* e^{-i\Delta k z}$$

$\Delta k = 2k_1 - k_2$

$$A_2' = \frac{1}{2} i \xi \frac{\omega_2}{n_2} A_1^2 e^{i\Delta k z}$$

note factor of $\frac{1}{2}$

It is important to understand limiting cases, esp. $\Delta k = 0$

1) SFG <u>upconversion</u>: assume $A_2' = 0$ (small A_1, A_3; large A_2) <u>undepleted</u>

$$A_1' = i \xi \frac{\omega_1}{n_1} A_2^* A_3$$

$$A_1'' = i \xi \frac{\omega_1}{n_1} A_2^* \left(i \xi \frac{\omega_3}{n_3} A_2 A_1 \right) = - \underbrace{\xi^2 |A_2|^2 \frac{\omega_1 \omega_3}{n_1 n_3}}_{\equiv K^2 > 0, \text{ real}} A_1$$

$\to$ osc solutions

$$A_1(z) = B \cos Kz + C \sin Kz$$

$$A_1' = -KB \sin Kz + KC \cos Kz = i \xi \frac{\omega_1}{n_1} A_2^* A_3$$

$$A_3(z) = -\frac{i K n_1}{\xi \omega_1 A_2^*} \left(-B \sin Kz + C \cos Kz \right)$$

valid for any $A_1(0), A_3(0)$ input

upconversion $A_3(0) = 0 \rightarrow C = 0 \qquad B = A_1(0)$

$$A_1(z) = A_1(0)\cos \kappa z$$

$$A_3(z) = \frac{i \kappa n_1}{\xi \omega_1 A_2^*} A_1(0)\sin \kappa z = i\,\frac{\xi |A_2| \left(\frac{\omega_1\omega_3}{n_1 n_3}\right)^{1/2}}{\xi (\omega_1/n_1) A_2^*} A_1(0)\sin \kappa z$$

$$= i \underbrace{\left(\frac{n_1\omega_3}{n_3\omega_1}\right)^{1/2}}_{} A_1(0)\, e^{i\phi_2} \sin \kappa z \qquad \text{let } A_2 = |A_2| e^{i\phi_2}$$

ampl. determines max conversion.

every photon at ω_3 uses a photon at ω_1

should have $n_1\frac{|A_1(z)|^2}{\omega_1} + n_3\frac{|A_3(z)|^2}{\omega_3} = n_1\frac{|A_1(0)|^2}{\omega_1}$

✓ $n_1\frac{|A_1(0)|^2}{\omega_1}\sin^2(\kappa z) + \frac{n_3}{\omega_3}\frac{n_1\omega_3}{n_3\omega_1}|A_1(0)|^2\cos^2 \kappa z = n_1\frac{|A_1|}{\omega_3}$

Note: > max conversion not set by d_{eff}

> but distance required to convert is $\propto 1/d_{eff}$

> If $A_1(0) = 0$ no output. In principle, vacuum fluct. can be converted, but getting some signal at ω_3 doesn't increase rate.

> down conversion possible too.

Application:

- convert IR light to visible to make use of available detectors.
- pulse at A_2 is a gate → time-resolved fluorescence.
- no gain in this process

2) DFG and parametric amp

here, assume strong pump at ω_3 $\quad A_3' = 0$ undepleted

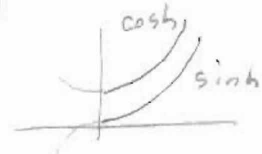
$$A_1' = i\xi \frac{\omega_1}{n_1} A_2^* A_3$$

$$A_2' = i\xi \frac{\omega_2}{n_2} A_1^* A_3 \rightarrow A_2'' = i\xi \frac{\omega_2}{n_2} A_3 \left(-i\xi \frac{\omega_1}{n_1} A_3^* A_2\right)$$

$$= +\underbrace{\xi |A_3|^2 \frac{\omega_1 \omega_2}{n_1 n_2}}_{K^2 > 0,\ \text{real}} A_2$$

now +'ve sign → exponential soln's.

- can seed with either one. (ω_1 or ω_2)
- exponential gain $\sinh(Kz)$ or $\cosh(Kz)$



for every photon we make at ω_1, get one at ω_2
- process builds on itself.

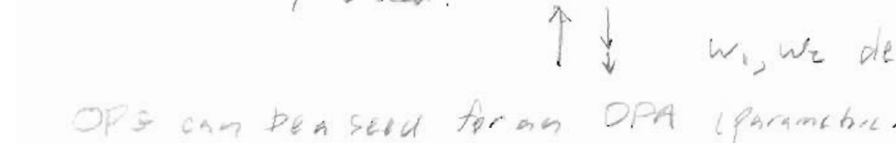
∴ high gain - can get optical parametric generation (OPG)
- also spontaneous param. down conversion

w/ no seed!

$\uparrow \downarrow$ $\quad \omega_1, \omega_2$ determined by $\Delta k = 0$

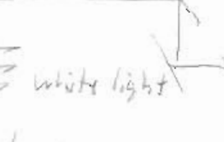
OPG can be a seed for an OPA (parametric amp)

- also use white-light from continuum.



can do multiple stages.

nomenclature



spontaneous parametric down conversion. only pump, no seed (vacuum seeding)
- used for creating "entangled" photons. for quantum optics.
- properties of photons in pair are linked.

3) second harmonic generation.

$$A_1' = i\xi \frac{\omega_1}{n_1} A_2 A_1^* e^{-i\Delta k z}$$

$$A_2' = \frac{1}{2} i \xi \frac{\omega_2}{n_2} A_1^2 e^{i\Delta k z}$$

this time do more to decouple eqns: scale A_1, A_2 to total intens.

$$I = I_1 + I_2 \rightarrow 2\varepsilon_0 c \left(n_1 |A_1|^2 + n_2 |A_2|^2 \right) = I$$

make amplitudes dimensionless

$$A_i = \left(\frac{I}{2n_i \varepsilon_0 c}\right)^{\frac{1}{2}} a_i \qquad \text{so that } |a_1|^2 + |a_2|^2 = 1$$

$$\left(\frac{I}{2n_2\varepsilon_0 c}\right)^{\frac{1}{2}} a_2' = i\frac{1}{2}\left(\frac{2d}{c}\right)\left(\frac{I}{2\varepsilon_0 n_1 c}\right) \frac{\omega_2}{n_2} a_1^2 e^{i\Delta k z}$$

remaining constants must have dims of $1/\ell$

let $\ell = \left(\frac{2\varepsilon_0 n_1^2 n_2 c}{I}\right)^{1/2} \frac{c}{2\omega_1 d_{eff}}$ eq 2.7.19 is wrong

- Note that $\omega_1 = \frac{1}{2}\omega_2$ in this case, so both eqn.s have the same ℓ.
- scale z to ℓ : $\xi = z/\ell$ $\qquad \Delta s \equiv \Delta k \ell$

$$a_1' = i a_1^* a_2 e^{-i\Delta s \xi}$$

$$a_2' = i a_1^2 e^{i\Delta s \xi}$$

From here

1) numeric solution (see mixing solutions.nb)

2) analytic:

let $a_i = u_i e^{i\phi_i}$ w/ u_i, ϕ_i real fcns.

separate out Re, Im parts of eqns.

for HW2

Scaled 3-wave mixing eqns. $I = I_1 + I_2 + I_3 = \text{const.}$

$$\left(\frac{2\pi I}{n_1 c}\right)^{\frac{1}{2}} \frac{da_1}{dz} = \frac{8\pi i d\omega_1}{n_1 c} \frac{2\pi I}{c\sqrt{n_2 n_3}} a_3 a_2^* e^{-i\Delta k z}$$

$$\frac{da_1}{dz} = i \frac{8\pi d\omega_1}{c} \sqrt{\frac{2\pi I}{n_1 n_2 n_3 c}} a_3 a_2^* e^{-i\Delta k z}$$

all are the same except for ω factor

define $\ell = \left(\frac{8\pi d\omega_3}{c}\sqrt{\frac{2\pi I}{n_1 n_2 n_3 c}}\right)^{-1}$ and $\xi = z/\ell$

$\Delta s = \Delta k \ell$

$$a_1' = i \frac{\omega_1}{\omega_3} a_3 a_2^* e^{-\Delta s \xi}$$

$$a_2' = i \frac{\omega_2}{\omega_3} a_3 a_1^* e^{-i\Delta s \xi}$$

$$a_3' = i \quad a_1 a_2 e^{+i\Delta s \xi}$$

Conservation Laws

photon energy:

$$\left(-\nabla^2 + \frac{n_3^2}{c^2}\frac{\partial^2}{\partial t^2}\right) E_3 e^{-i\omega_3 t} = 4\pi \chi^{(2)} E_1 e^{-i\omega_1 t} E_2 e^{-i \dots}$$

to cancel time-dep. exponentials,

$$\hbar\omega_3 = \hbar\omega_1 + \hbar\omega_2$$

photon momentum

same argument if k's are all parallel

$$\hbar k_3 = \hbar k_1 + \hbar k_2$$

note that $\hbar k_i = \hbar n(\omega_i)\frac{\omega}{c}$

if $\omega_1 = \omega_2$,

$$\hbar k_3 = 2\hbar k_1$$

or $\quad \frac{n(\omega_3)\omega_3}{c} = 2n(\omega_1)\frac{\omega_1}{c} \rightarrow \frac{1}{v_{ph}(\omega_3)} = \frac{1}{v_{ph}(\omega_1)}$

so phase vel. are equal if $\Delta k = 0$

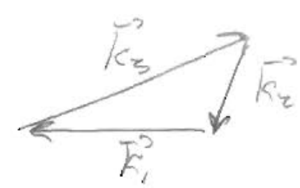
but for SFG this isn't right

$$\frac{\omega_3}{v_{ph3}} = \frac{\omega_1}{v_{ph1}} + \frac{\omega_2}{v_{ph2}}$$

so just remember $\Delta k = \sum k_i$

noncollinear case:

$$\vec{k}_3 = \vec{k}_1 + \vec{k}_2$$



beam power: Manley - Rowe

$$\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right) = \frac{d}{dz}\left(\frac{I_2}{\omega_2}\right) = -\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right)$$

intensity is a p[illegible] flux $\quad I_i = \frac{1}{2} v_i \cdot U_i = \frac{1}{2} \frac{c}{n_i} \frac{n_i^2 \mathcal{E}_i^{o2}}{8\pi}$

note location of index

$$I_i = \frac{\hbar\omega_i \cdot N_i}{\text{area} \cdot \text{time}} = \hbar\omega_i F_i$$

$\rightarrow$ photon flux.

$\therefore$ see that M-R relations result from photon conservation.