**MATH 225 - Differential Equations** June 16 , 2008

**Homework 9, Field 2008** **Due Date**: June 17, 2008

STABLE COMPETITION - COMPLETIVE EXCLUSION

Today in class we studied the nonlinear system of differential equations,

$$\frac{dx}{dt} = x(10 - x - y) \quad (1)$$

$$\frac{dy}{dt} = y(30 - 2x - y), \quad (2)$$

and found out that there existed four equilibrium points and that the two-species physical solutions (solutions in the first quadrant of phase space) tended to the a real sink at $(x, y) = (0, 30)$. We say that this competition is unstable in the sense that in competitive situations all $x$ trajectories tend to zero as $t$ goes to infinity.

Suppose now that we have the following system of differential equations,

$$\frac{dx}{dt} = x(2 - 2x - y) \quad (3)$$

$$\frac{dy}{dt} = y(2 - x - 2y), \quad (4)$$

where $x, y \in [0, \infty)$, which models competition in two species $x$ and $y$.

1. Find all fixed points of the system.

2. Using the Jacobian matrix classify these fixed points.

3. Do a full eigenvalue/eigenvector decomposition of the off-axis equilibrium and using this approximate solution discuss the local behavior of the system in a neighborhood of this equilibrium.

4. Using HPGSYSTEMSOLVER plot the slope field in the first quadrant and plot enough trajectories to give a qualitative description of the time-dynamics of the populations. Is the system stable in the sense that there exists an equilibrium population such that both $x$ and $y$ are nonzero?

5. Using HPGSYSTEMSOLVER plot the slope field of the system of differential equations,

$$\frac{dx}{dt} = x(2 - x - 2y) \quad (5)$$

$$\frac{dy}{dt} = y(2 - 2x - y), \quad (6)$$

   where $x, y \in [0, \infty)$, which models competition in two species $x$ and $y$. Compare the time dynamics of this model to the model (3)-(4) and discuss the differences in stability.