MATH348 - October 13, 2011 **NAME**:
Exam I - 50 Points

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points)

   (a) True/False: No Justification Needed

   i. The function e^{ix} has even symmetry.

   ii. If a periodic function is neither even nor odd then its Fourier series representation must have sine terms/modes.

   iii. If the complex Fourier coefficients are purely real then the periodic function is even.

   iv. A function can have only one periodic extension.

   v. A truncated Fourier sine half-range expansion will not have Gibb's phenomenon.

   (b) Short Response

   i. Provide two physical interpretations of both Fourier coefficients and their corresponding Fourier modes.

   ii. Explain Gibb's phenomenon. What is it and when can you expect it to occur?

2. (10 Points)

(a) Given that n is an integer for following integrals,

$$\int_{-\pi}^{\pi} f(x)dx = \left.\frac{x^3}{3}\right|_{-\pi}^{\pi},$$

$$\int_{-\pi}^{\pi} f(x)\cos(nx)dx = \left.\left[\frac{x^2\sin(nx)}{n} + \frac{2x\cos(nx)}{n^2} - \frac{2\sin(nx)}{n^3}\right]\right|_{-\pi}^{\pi},$$

$$\int_{-\pi}^{\pi} f(x)\sin(nx)dx = \left.\left[\frac{x^2\cos(nx)}{n} + \frac{2x\sin(nx)}{n^2} - \frac{2\cos(nx)}{n^3}\right]\right|_{-\pi}^{\pi},$$

$$\int_{-\pi}^{\pi} g(x)dx = e^{-i\pi} - e^{i\pi},$$

$$\int_{-\pi}^{\pi} g(x)e^{-inx}dx = \left.e^{inx}\left(\frac{1}{n^2} - \frac{ix}{n}\right)\right|_{-\pi}^{\pi}.$$

i. Calculate the symmetry and Fourier series of $f(x)$.

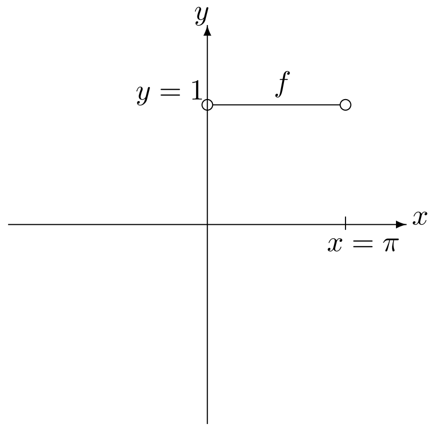
ii. Calculate the symmetry and Fourier series of $g(x)$.

iii. From the complex Fourier series of g calculate the corresponding real Fourier series.

(b) The following table contains different boundary conditions for the ODE, $F'' + \lambda F = 0$, $\lambda \in [0, \infty)$. Fill in each table element with either a yes or a no.

	Boundary value problem has a cosine solution	Boundary value problem has a sine solution	Boundary value problem has a nontrivial constant solution
$F(0) = 0,\ F(L) = 0$			
$F(0) = 0,\ F'(L) = 0$			
$F'(0) = 0,\ F'(L) = 0$			
$F'(0) = 0,\ F(L) = 0$			

3. (10 Points) Suppose f is given by the graph below.



(a) On the graph above, sketch the Fourier cosine <u>and</u> sine half-range expansions with dashed lines and solid lines, respectively.

(b) Find the Fourier coefficients of the cosine and sine half-range expansions. Justify your calculations.

4. (10 Points) Find the complex Fourier series representation of

$$f(x) = \begin{cases} x, & -1 < x < 0, \\ 0, & 0 < x < 1. \end{cases}$$

5. (10 Points) Given,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad \begin{matrix} x \in (0,\pi) \\ t \in (0,\infty) \end{matrix}, \tag{1}$$

$$u_x(0,t) = 0,\ u_x(\pi,t) = 0, \tag{2}$$

$$u(x,0) = f(x). \tag{3}$$

(a) Describe the separation of variable procedure used to solve the partial differential equation given by (1)-(3). Be sure to discuss how each step corresponds to each equation (1), (2) and (3).

(b) The separation of variables process, applied to (1), yields the equation

$$F''(x) + \lambda F(x) = 0, \quad \lambda \in \mathbb{R}. \tag{4}$$

Find all nontrivial solutions of (4) that satisfies (2). Justify your choices.