Note Title 2/19/2010

Schedule: - Monday practice exam posted on wiki, work in your tablet pairs in class.
- Tuesday solns to practice exam & homework posted on the wiki.
- Wednesday hand in homework & take exam 2.

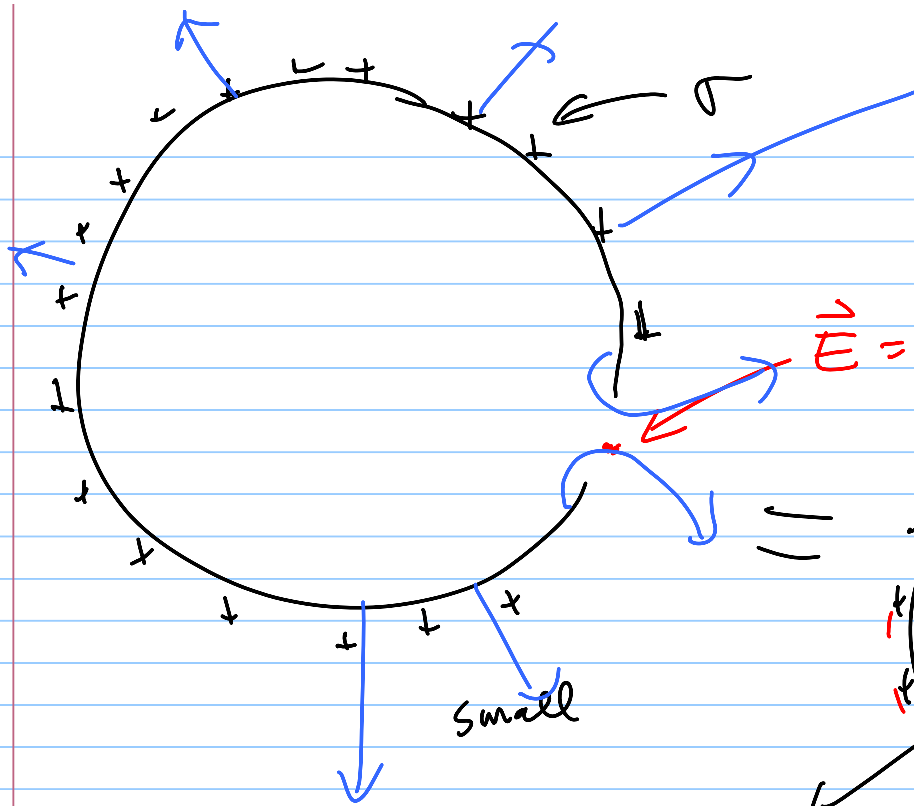
Exam 2: If you got a 7 or above on problems 1 & 3 of exam 1 there will be 2 problems. If you have not contacted me about your score of 6 or below on these problems then there will be either 1 or 2 additional problems on exam 2 for you to complete. These extra problems will be given to you at a later date by contacting me.

Exam 2 learning objectives

- be able to apply separation of varibles to a partial diff. eqn (PDE) from 1st principles
- understand that the sep. variables soln is not a general solution to the PDE
- be able to apply the superposition principle, along with the orthogonality property, to form a solution which satisfies arbitrary boundary conditions.

methods - Coul law

Superposition



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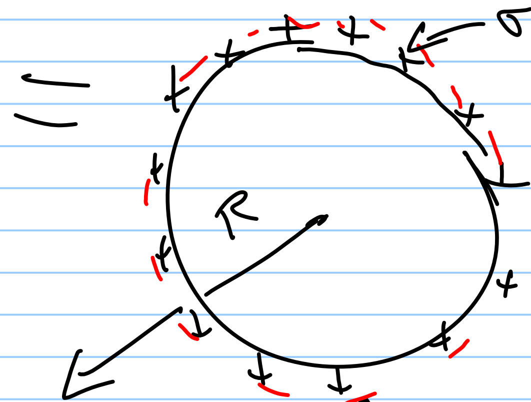

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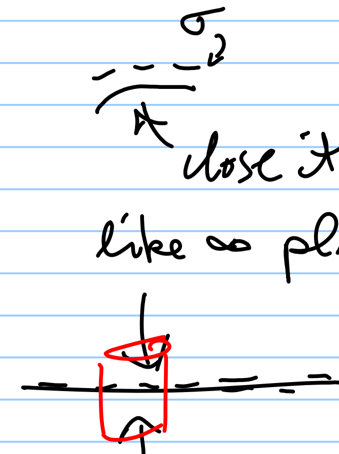


close it looks like ∞ plane

upper cap

$$-EA - EA = \frac{\sigma A}{\epsilon_0}$$

$$E = -\frac{\sigma}{2\epsilon_0}\hat{r}$$

E surface
||

$$E\,4\pi R^2 = \frac{\sigma\,4\pi R^2}{\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{r}$$

$$\bar{E}_{net} = \frac{\sigma}{\epsilon_0}\hat{r} - \frac{\sigma}{2\epsilon_0}\hat{r}$$

$$= \frac{\sigma}{2\epsilon_0}\hat{r}$$

$$V(r,\theta,\varphi) = R(r)\,\Theta(\theta)\,\Phi(\varphi)$$

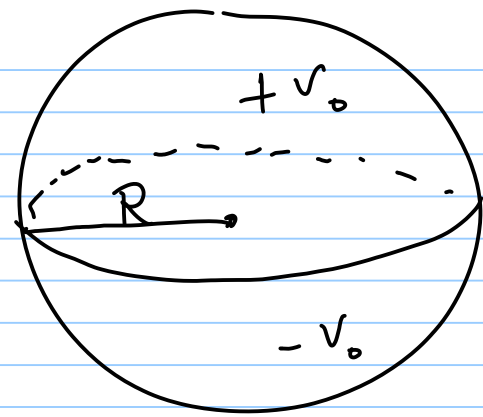
$$V(r,\theta) = \sum_{\ell=0}^{\infty} \left[A_\ell r^\ell + B_\ell r^{-(\ell+1)} \right] P_\ell(\cos\theta)$$

$$\int_{-1}^{1} P_\ell(x) P_m(x)\,dx = \int_{0}^{\pi} P_\ell(\cos\theta) P_m(\cos\theta) \sin\theta\,d\theta \begin{cases} 0 & \text{if } \ell \neq m \\ \dfrac{2}{2m+1} & \text{if } \ell = m \end{cases}$$

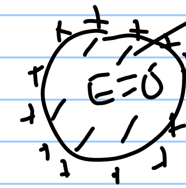
or $\dfrac{2}{2m+1}\,\delta_{\ell m}$

$x = \cos\theta \qquad dx = -\sin\theta\,d\theta$

Ex:

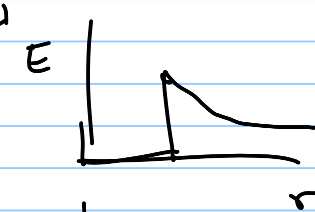


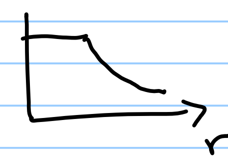
$$\vec{E} = -\vec{\nabla} V$$



conductor

$E = \frac{\sigma}{\epsilon_0}$ near surface





$$V = \sum_{\ell}{}' \left[A_\ell' r^\ell + B_\ell' r^{-(\ell+1)}\right] P_\ell$$

V continuous but not

For $r < R$ $B_\ell' = 0$ since V must be finite at $r = 0$

For $r > R$ $A_\ell' = 0$ " " " $\rightarrow 0$ as $r \rightarrow \infty$

Thus $V_{inside} = \sum A_\ell' r^\ell P_\ell$ $V_{outside} = \sum B_\ell' r^{-(\ell+1)} P_\ell$

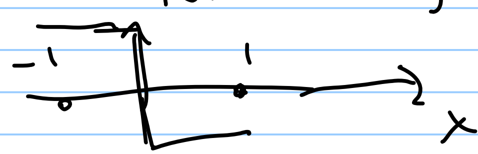
at $r = R$ $\quad V_{in}\big|_R = V_{out}\big|_R$

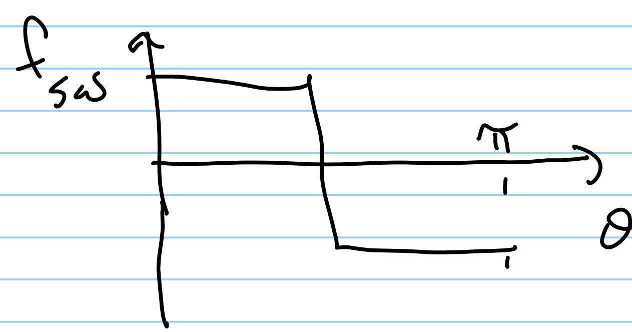
$$A'_\ell R^\ell P_\ell = B'_\ell R^{-(\ell+1)} P_\ell$$

$$A'_\ell R_\ell = B'_\ell R^{-(\ell+1)} \equiv A_\ell$$

$$\Rightarrow \quad A'_\ell = A_\ell / R^\ell \quad \& \quad B'_\ell = A_\ell / R^{-(\ell+1)} = A_\ell R^{(\ell+1)}$$

$$V_{in} = \sum A_\ell \left(\frac{r}{R}\right)^\ell P_\ell \quad ; \quad V_{out} = \sum_\ell A_\ell \left(\frac{R}{r}\right)^{\ell+1} P_\ell$$





$x = \cos\theta$ $\quad \theta : 0 \to \pi$

$x : 1 \to -1$

Define square wave function $f_{sw}(\theta) = \begin{cases} +1 & 0<\theta<\pi/2 \\ -1 & \pi/2<\theta<\pi \end{cases}$

at $r = R$ $\quad V_{in}(R) = \sum_l A_l P_l$ $\quad$ multiply by $P_m(\cos\theta)$, & integrate

$$\int_{-1}^{1} V_0 f_{sw} P_m \, d(\cos\theta) = \sum_l A_l \int_{-1}^{1} P_l P_m \, d(\cos\theta)$$

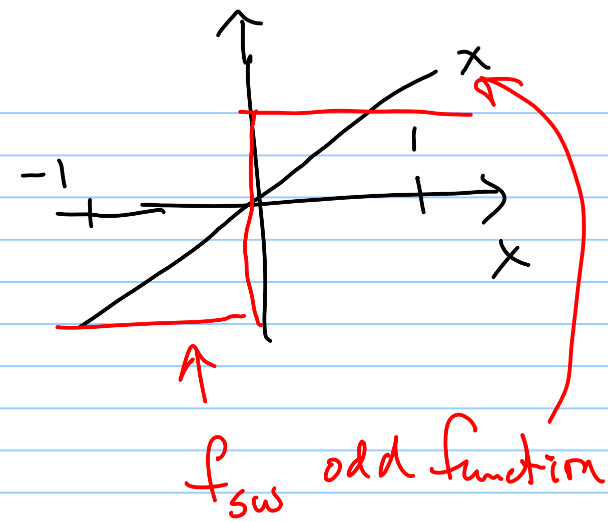
$$= \sum_{l=0}^{\infty} A_l \frac{2\,\delta_{lm}}{2l+1} = \frac{2A_m}{2m+1}$$

$$A_m = \frac{2m+1}{2} V_0 \int_{-1}^{1} f_{sw} P_m(\underbrace{\cos\theta}_{x}) \, d(\underbrace{\cos\theta}_{x})$$

$P_0 = 1 \quad P_1 = x = \cos\theta \quad P_2 = \frac{1}{2}(3\cos^2\theta - 1) \quad P_3 = \frac{1}{2}(5x^3 - 3x)$

$$A_1 = \frac{3}{2} V_0 \int_{-1}^{1} f_{sw} \underbrace{\cos\theta}_{x} \underbrace{d(\cos\theta)}_{dx}$$

odd function



$x = \cos\theta \qquad \theta : 0 \to \pi$

$\cos\theta : 1 \to -1$

$$A_1 = \frac{3}{2} V_0 \int_0^1 1\, x\, dx + \frac{3}{2} V_0 \int_0^1 1\, x\, dx$$