Homework 8 due 27 Oct. 2006 by 5pm
PH462 EM Waves and Optical Physics posted: 23 Oct. 2006

1) HM problem 9.4

2) HM problem 9-12.

3) HM problem 9-16. This impedence-matching problem is not specific to antennas – it works for any load resistance.

4) HM problem 9-17: You can consider the rotating dipole as a superposition of two linear dipoles oriented along the x and y axes.

5) When the dipole radiates because it is driven by an external wave, the incident radiation is *scattered*. We can use this description of scattering when the scattering object is much less than the wavelength.

a. A forced, damped oscillator has a position dependence:

$$x(t) = \frac{-eE_0}{m_e}\frac{e^{-i\omega t}}{\left(\omega_0^2 - \omega^2\right) - i\nu\omega} ,$$

where ω_0 is the resonance frequency, and ν is the radiative damping rate for a particular resonance (see class notes for the radiation damping of a freely oscillating charge). Calculate the total radiated power, using the Larmor equation (eq. 8-89 in HM). To calculate the acceleration, do the time derivatives, then take the real part, that is $\left\langle a^2 \right\rangle = \left\langle (\mathrm{Re}[\ddot{x}])^2 \right\rangle$.

b. The scattering cross-section σ is calculated by dividing the radiated power by the incident intensity: $\sigma = P_{avg} / I_{inc}$. Show that the scattering cross-section is given by:

$$\sigma = \frac{8\pi r_e^2}{3}\frac{\omega^4}{\left(\omega_0^2 - \omega^2\right)^2 + (\nu\omega)^2}$$, where the classical electron radius is $r_e = e^2 / m_e c^2$.

c. Show that in the low frequency limit, i.e. $\omega << \omega_0$, $\nu << \omega_0$, $\sigma = \frac{8\pi r_e^2}{3}\left(\frac{\omega}{\omega_0}\right)^4$.

In this case, we have Rayleigh scattering, with the characteristic ω^4 dependence. (One reason why the sky is blue.)

d. Show that in the high frequency limit, i.e. $\omega >> \omega_0$, $\sigma = \frac{8\pi r_e^2}{3}$. This is the limit of Thomson scattering, the scattering from a free electron.

e. Make a plot of $\mathrm{Log}_{10}[\sigma]$ vs ω/ω_0 for a range near $0 < \omega/\omega_0 < 3$, for two damping rates, $\nu = 0.1\ \omega_0$ and $\nu = 0.02\ \omega_0$.