MACS 332 - August 9, 2006 **NAME:**
Exam III - 60 minutes

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. Please enclose your final answers in boxes.

1. (10 Points) Briefly describe the following mathematical concepts.

a. The norm of a vector in $\mathbb{R}^n$.

b.The Gram-Schmidt Process.

c. The general least-squares problem.

d. The matrix $\mathbf{U}_{n\times n}$ where $\mathbf{U}$ is such that $\mathbf{U}^{\mathrm{T}}\mathbf{U}=\mathbf{U}\mathbf{U}^{\mathrm{T}}=\mathbf{I}$.

2. (10 Points) Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

a. Determine the least-squares solution of $\mathbf{Ax}{=}\mathbf{b}$.

b. Determine the least-squares error associated with the least-squares solution in (**a**).

3. (10 Points) Let

$$\mathbf{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$

Calculate the distance from $\mathbf{y}$ to the plane W=span$\{\mathbf{u}_1,\mathbf{u}_2\}$.

4. (10 Points) **Note** - Problem 4b is on page 4.

a. Let $Q(\mathbf{x})=\mathbf{x}^T\mathbf{A}\mathbf{x}$, where $\mathbf{A}=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Determine the change of variable matrix $\mathbf{P}$ and diagonal matrix $\mathbf{D}$ such that $Q(\mathbf{x})=Q(\mathbf{y})=\mathbf{y}^T\mathbf{D}\mathbf{y}$, where $\mathbf{y}=\mathbf{P}^T\mathbf{x}$.

b. Let Q($\mathbf{x}$)$= 3x_2^2 - 4x_1x_2 + 6x_1^2$. Determine the symmetric two-by-two matrix $\mathbf{A}$, such that Q($\mathbf{x}$)=$\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x}$.

5. (10 Points) Let $\mathbf{A}=\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix}$. Determine the matrices $\mathbf{U}$,Σ,$\mathbf{V}$, associated with $\mathbf{A}=\mathbf{U}\Sigma\mathbf{V}^{\mathrm{T}}$.