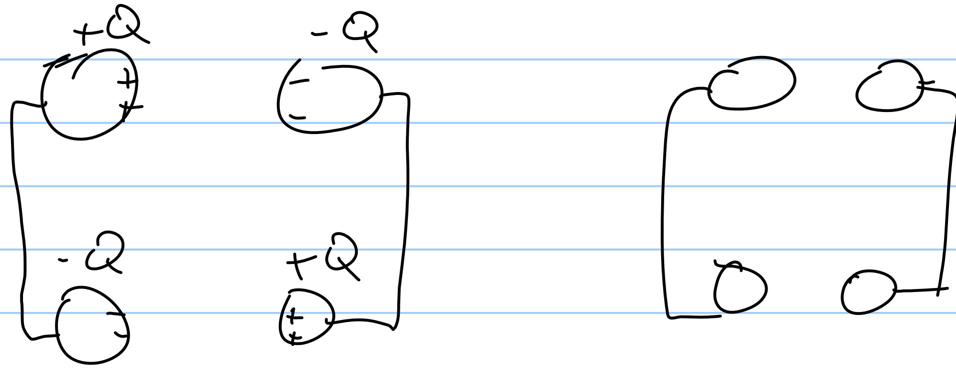
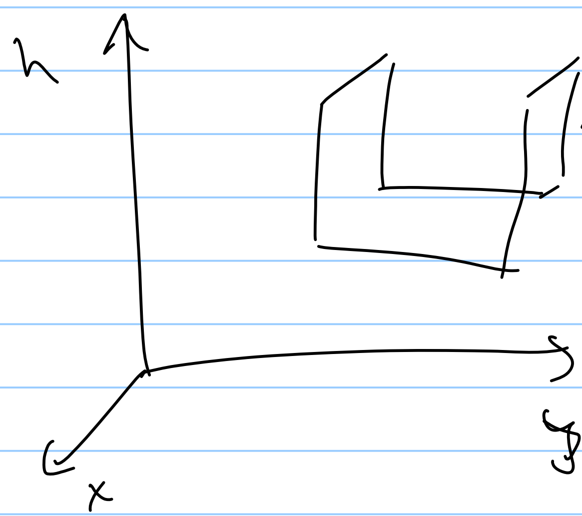
Note Title 2/13/2009

Uniqueness Theorem: find a soln to Laplace's eqn & obeys bndry cond. Then it is the soln.

4 conductors



Laplaces eqn satisfied film on soap



← wire dipped into soap

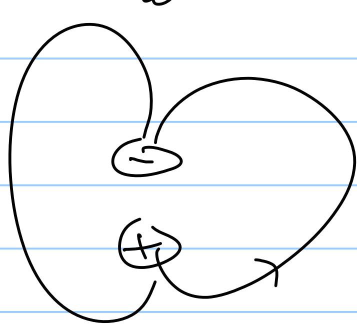
$$\nabla^2 h(x,y) = 0$$

& bndry cond.

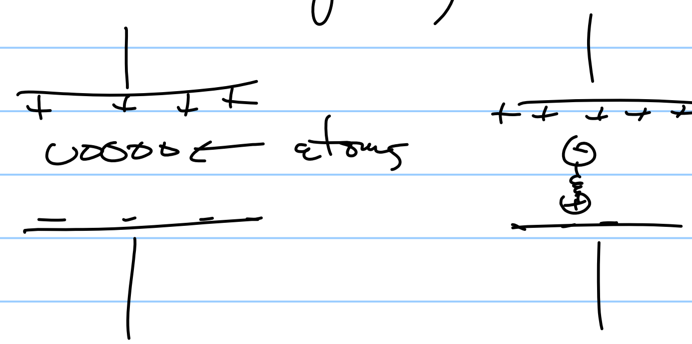
$h(x,y)\Big|_{bndry}$ = on the wire

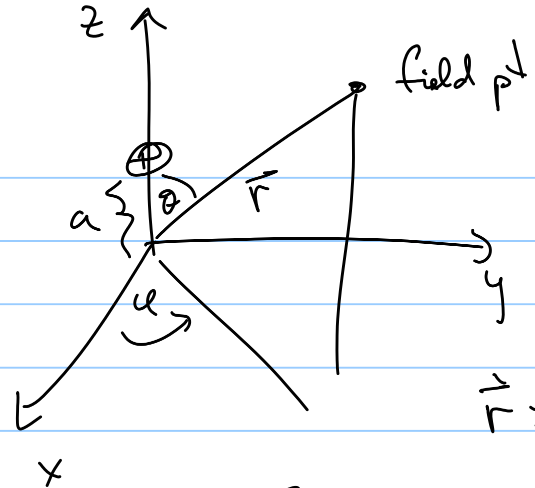
Deal with material (fields in glass)

- atomic theory



dipole field





$$V(r,\vartheta) = ? \quad \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' \qquad \vec{r}' = a\hat{z}$$

$$\vec{r} = r\sin\theta\cos\varphi\,\hat{x} + r\sin\theta\sin\varphi\,\hat{y} + r\cos\theta\,\hat{z}$$

$$|\vec{\mathscr{r}}| = \left[r^2\sin^2\theta\cos^2\varphi + r^2\sin^2\theta\sin^2\varphi + (-a + r\cos\theta)^2\right]$$

$$V(r,\vartheta) = \frac{q}{4\pi\epsilon_0}\frac{1}{(r^2+a^2-2ar\cos\theta)^{1/2}}$$

$r > a$
$$r\left(\frac{r^2}{r^2} + \frac{a^2}{r^2} - \frac{2ar\cos\vartheta}{r^2}\right)^{1/2}$$

$$= r\Big(1 + \underbrace{\frac{a^2}{r^2} - \frac{2a}{r}\cos\theta}_{\epsilon}\Big)^{1/2} = r\sqrt{1+\epsilon}$$

taylor expansion (binomial)

$$\frac{1}{r(1+\epsilon)^{1/2}} = \frac{1}{r}\left[1 - \frac{1}{2}\left(\frac{a^2}{r^2} - \frac{2a}{r}\cos\vartheta\right) + \frac{3}{8}\left(\frac{a^2}{r^2} - \frac{2a}{r}\cos\vartheta\right)^2 + \cdots\right]$$

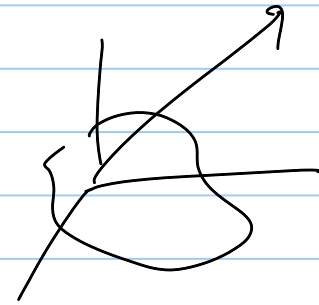
arrange in powers $\frac{a}{r} \ll 1$

$$\frac{1}{r(1+\epsilon)^{1/2}} = \frac{1}{r}\left\{1 + \frac{a}{r}\cos\vartheta + \left(\frac{a}{r}\right)^2\left[\frac{3\cos^2\vartheta - 1}{2}\right]\right\} + \cdots$$

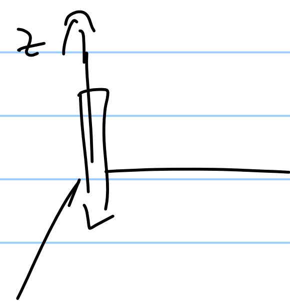
P_ℓ

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{l=0}^{\infty} \left(\frac{a}{r}\right)^l P_l(\cos\theta) \qquad r > a$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')\,d\tau'}{\mathscr{r}}$$



Axial multip mom expansion (charges only on z axis



$$\rho(\vec{r}\,') = \delta(x')\,\delta(y')\,\lambda(z')$$

$$d\tau' = dx'\,dy'\,dz'$$

$$\int \delta(x')\,dx' = 1$$

$$V = \frac{1}{4\pi\epsilon_0}\int \lambda(z') \; \frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{z'}{r}\right)^l P_l(\cos\theta)\, dz'$$

$$= \frac{1}{4\pi\epsilon_0}\sum \frac{M_l\, P_l(\cos\theta)}{r^{l+1}} \qquad \text{where } M_l = \int \lambda(z')(z')^l\, dz'$$

axial multipole moments

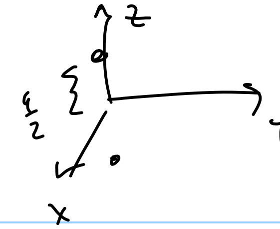
$l = 0 \qquad \int \lambda(z')(z')^0\, dz' = Q$

monopole $l=0$ $\quad V \propto \frac{1}{r}$

dipole $l=1$ $\quad V \propto \frac{1}{r^2}$

quad $l=3$ $\quad V \propto 1/r^4$

$$\lambda(z') = q_0\,\delta\left(z'-\frac{a}{2}\right) - q_0\,\delta\left(z'+\frac{a}{2}\right)$$



find M_l

$$M_l = \int \lambda(z')(z')^l\,dz' = \int_{-\infty}^{\infty} q_0\,\delta\left(z'-\frac{a}{2}\right)(z')^l\,dz' - \int q_0\,\delta\left(z'+\frac{a}{2}\right)(z')^l\,dz'$$

$z' = \frac{a}{2}$

$z' = -a/2$

$$= q_0\left(\frac{a}{2}\right)^l - q_0\left(-\frac{a}{2}\right)^l$$

$l=1$: $q_0\left(\frac{a}{2}\right) - q_0\left(-\frac{a}{2}\right) = q_0 a$ dipole

$l=2$: $q_0\left(\frac{a}{2}\right)^2 - q_0\left(-\frac{a}{2}\right)^2$ quad.

$$\int f(x)\,\delta\left(x-\frac{a}{2}\right)dx = f\left(\frac{a}{2}\right)$$