MATH 235 - SPRING 2008

HOMEWORK 1

[1] EVALUATE THE FOLLOWING INTEGRALS.

(a) $\int x^3\cos(5x)dx$

$$\begin{array}{rc|l}
 & u & dv \\ \hline
+ & x^3 & \cos(5x) \\
- & 3x^2 & \searrow \frac{1}{5}\sin(5x) \\
+ & 6x & \searrow \frac{-1}{25}\cos(5x) \\
- & 6 & \searrow \frac{-1}{125}\sin(5x) \\
+ & 0 & \searrow \frac{1}{625}\cos(5x)
\end{array}$$

$$= \boxed{\frac{x^3}{5}\sin(5x) + \frac{3x}{25}\cos(5x) - \frac{6x}{125}\sin(5x) - \frac{6}{625}\cos(5x) + C}$$

(b) $\int x^2\sin(2x^3)dx$

$u = 2x^3$

$du = 6x^2dx$

$= \frac{1}{6}\int \sin(u)\,du$

$= -\frac{1}{6}\cos(u) + C = \boxed{-\frac{1}{6}\cos(2x^3) + C}$

(c) $\int \frac{x^2}{x^2+1}dx$

$$\begin{array}{r|l}
 & 1 - \frac{1}{x^2+1} \\ \hline
x^2+1 & x^2+0 \\
 & -(x^2+1) \\ \hline
 & -1
\end{array}$$

$= \int\left[1 - \frac{1}{x^2+1}\right]dx$

$= \boxed{x - \tan^{-1}(x) + C}$

(d) $\int \frac{4-2x}{(x^2+1)(x-1)^2}dx = \int \frac{Ax+B}{x^2+1} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2}dx$

$(Ax+B)(x-1)^2 + C(x-1)(x^2+1) + D(x^2+1) = 4-2x$

$Ax^3-2Ax^2+Ax+Bx^2-2Bx+B+Cx^3-Cx^2+Cx-C+Dx^2+D = 4-2x$

$(A+C)x^3+(-2A+B-C+D)x^2+(A-2B+C)x+(B-C+D) = 4-2x$

$$\begin{array}{ll} A+C=0 & A=2 \\ -2A+B-C+D=0 & B=1 \\ A-2B+C=-2 & C=-2 \\ B-C+D=4 & D=1 \end{array}$$

$$=\int \frac{2x+1}{x^2+1}+\frac{-2}{x-1}+\frac{1}{(x-1)^2}\,dx$$

$$=\int \frac{2x}{x^2+1}+\frac{1}{x^2+1}-\frac{2}{x-1}+\frac{1}{(x-1)^2}\,dx$$

$$=\boxed{\ln|x^2+1|+\arctan|x|-2\ln|x-1|-\frac{1}{x-1}+C}$$

(e) $\int \frac{5x}{3x-1}\,dx \qquad u=3x-1 \Rightarrow x=\frac{u+1}{3}$

$\qquad du=3\,dx$

$$=\frac{1}{3}\int \frac{5\left(\frac{u+1}{3}\right)}{u}\,du=\frac{5}{9}\int \frac{u+1}{u}\,du$$

$$=\frac{5}{9}\int\left(\frac{u}{u}+\frac{1}{u}\right)du=\frac{5}{9}\left[u+\ln|u|\right]+C$$

$$=\boxed{\frac{5}{9}\left[3x-1+\ln|3x-1|\right]+C}$$

$\boxed{2}$ ASSUMING THAT $s\in\mathbb{R}$, EVALUATE THE FOLLOWING IMPROPER INTEGRALS.

(a) $\int_0^\infty x^3 e^{\beta t}e^{-st}\,dt$ WHERE $\beta\in\mathbb{R}$ AND $s>\beta$

$$=\frac{x^3e^{(\beta-s)t}}{\beta-s}\Big|_0^\infty=\lim_{t\to\infty}\frac{x^3e^{(\beta-s)t}}{\beta-s}-\frac{x^3e^{(\beta-s)(0)}}{\beta-s}$$

$$=0-\frac{x^3}{\beta-s}=\boxed{\frac{x^3}{\beta-s}}$$

MATH 235 - SPRING 2008

HOMEWORK 1

(b) $\int_0^\infty e^{-st}\cos(wt)\,dt$ WHERE $w \in \mathbb{R}$ AND $s > 0$

$$\begin{array}{c|c|c} & u & dv \\ \hline + & e^{-st} & \cos(wt) \\ - & -se^{-st} \searrow & \frac{1}{w}\sin(wt) \\ + & s^2e^{-st} \searrow\rightarrow & \frac{-1}{w^2}\cos(wt) \end{array}$$

$$\Rightarrow \int_0^\infty e^{-st}\cos(wt)\,dt = \frac{e^{-st}}{w}\sin(wt) - \frac{se^{-st}}{w^2}\cos(wt) - \frac{s^2}{w^2}\int_0^\infty e^{-st}\cos(wt)\,dt$$

$$+\frac{s^2}{w^2}\int_0^\infty e^{-st}\cos(wt)\,dt \qquad\qquad +\frac{s^2}{w^2}\int_0^\infty e^{-st}\cos(wt)\,dt$$

$$\Rightarrow \left(1+\frac{s^2}{w^2}\right)\int_0^\infty e^{-st}\cos(wt)\,dt = \frac{e^{-st}}{w}\sin(wt) - \frac{se^{-st}}{w^2}\cos(wt)\Big|_0^\infty$$

$$\Rightarrow \int_0^\infty e^{-st}\cos(wt)\,dt = \frac{w^2}{w^2+s^2}\left[\frac{e^{-st}}{w}\sin(wt) - \frac{se^{-st}}{w^2}\cos(wt)\right]_0^\infty$$

$$= \lim_{t\to\infty}\left[\frac{w^2}{w^2+s^2}\left\{\frac{e^{-st}}{w}\sin(wt) - \frac{se^{-st}}{w^2}\cos(wt)\right\}\right] - \left[\frac{w^2}{w^2+s^2}\left(\frac{s}{w^2}\right)\right]$$

$$= 0 + \frac{s}{w^2+s^2} = \boxed{\frac{s}{w^2+s^2}}$$

(c) $\int_0^\infty e^{-st}\sin(wt)\,dt$ WHERE $w \in \mathbb{R}$ AND $s > 0$

$$\begin{array}{c|c|c} & u & dv \\ \hline + & e^{-st} \searrow & \sin(wt) \\ - & -se^{-st} \searrow & \frac{-1}{w}\cos(wt) \\ + & s^2e^{-st} \rightarrow & \frac{-1}{w^2}\sin(wt) \end{array}$$

$$\int_0^\infty e^{-st}\sin(wt)\,dt = \frac{-e^{-st}}{w}\cos(wt) - \frac{se^{-st}}{w^2}\sin(wt) - \frac{s^2}{w^2}\int_0^\infty e^{-st}\sin(wt)\,dt$$

$$+\frac{s^2}{w^2}\int_0^\infty e^{-st}\sin(wt)\,dt \qquad\qquad +\frac{s^2}{w^2}\int_0^\infty e^{-st}\sin(wt)\,dt$$

$$\left(1+\frac{s^2}{w^2}\right)\int_0^\infty e^{-st}\sin(wt)\,dt = \left[\frac{-e^{-st}}{w}\cos(wt) - \frac{se^{-st}}{w^2}\sin(wt)\right]_0^\infty$$

$$= \frac{w^2}{w^2+s^2}\left[\frac{-e^{-st}}{w}\cos(wt) - \frac{se^{-st}}{w^2}\sin(wt)\right]_0^\infty$$

$$= \lim_{t\to\infty}\left[\frac{w^2}{w^2+s^2}\left(\frac{-e^{-st}}{w}\cos(wt) - \frac{se^{-st}}{w^2}\sin(wt)\right)\right] - \left[\frac{w^2}{w^2+s^2}\left(\frac{-1}{w}\right)\right]$$

$$= 0 + \frac{w}{w^2+s^2} = \boxed{\frac{w^2}{w^2+s^2}}$$

$\boxed{3}$ SOLVE THE FOLLOWING EQUATIONS FOR X.

(a) $\ln(x) - \ln(x-4) = -13$

$\Rightarrow \ln\left(\frac{x}{x-4}\right) = -13$

$\Rightarrow e^{\ln\left(\frac{x}{x-4}\right)} = e^{-13}$

$\Rightarrow \frac{x}{x-4} = e^{-13}$

$\Rightarrow x = e^{-13}(x-4)$

$\Rightarrow x - xe^{-13} = -4e^{-13}$

$\Rightarrow \boxed{x = \frac{-4e^{-13}}{1-e^{-13}}}$

(b) $\ln(x) + \ln(x-4) = -13$

$\Rightarrow \ln(x(x-4)) = -13$

$\Rightarrow e^{\ln(x(x-4))} = e^{-13}$

$\Rightarrow x(x-4) = e^{-13}$

$\Rightarrow x^2 - 4x - e^{-13} = 0$

$\boxed{x = \frac{4 \pm \sqrt{16+4e^{-13}}}{2}} \approx 0, 4$

(c) $e^{2(\ln(x)-\ln(x^2))}=1$

$\Rightarrow e^{2(\ln(\frac{x}{x^2}))}=1$

$\Rightarrow e^{\ln((\frac{x}{x^2})^2)}=1$

$\Rightarrow \frac{x^2}{x^4}=1 \Rightarrow x^2=1$

$\boxed{x=\pm 1}$

$\boxed{4}$ FIND ALL POINTS (x,y) WHICH SOLVE THE SIMULTANEOUS SYSTEMS.

(a) $$\begin{aligned} 4x-7y&=1 \\ 3x+6y&=1 \end{aligned} \Rightarrow \begin{array}{r} 24x-42y=6 \\ \underline{+21x+42y=7} \\ 45x \quad =13 \end{array} \Rightarrow x=\frac{13}{45}$$

BACK SUBSTITUTING:

$4(\frac{13}{45})-7y=1 \Rightarrow y=\frac{1}{45}$ $\qquad \boxed{(\frac{13}{45},\frac{1}{45})}$

(b) $2x-\frac{2x^2}{3}-xy=0$

$4xy-16y=0$

IF $x=y=0$ THEN THE EQUATIONS ARE TRUE AND $(0,0)$ IS A SOLUTION

IF $x\neq 0, y\neq 0$:

$$\Rightarrow \begin{aligned} 2-\tfrac{2}{3}x-y&=0 \\ 4x-16&=0 \end{aligned} \Rightarrow 4x=16 \Rightarrow x=4$$

BACK SUBSTITUTING:

$2-\frac{2}{3}(4)-y=0 \Rightarrow y=-\frac{2}{3}$

THE SOLUTIONS ARE $\boxed{(0,0),(4 -\frac{2}{3})}$

MATH 235 - SPRING 2008

HOMEWORK 1

(c) $yx^2+y^3-y=0$

$x-x^3-xy^2=0$

$\Rightarrow x^2+y^2-1=0$

$1-x^2-y^2=0$

$\Rightarrow x^2+y^2=1$

$x^2+y^2=1$

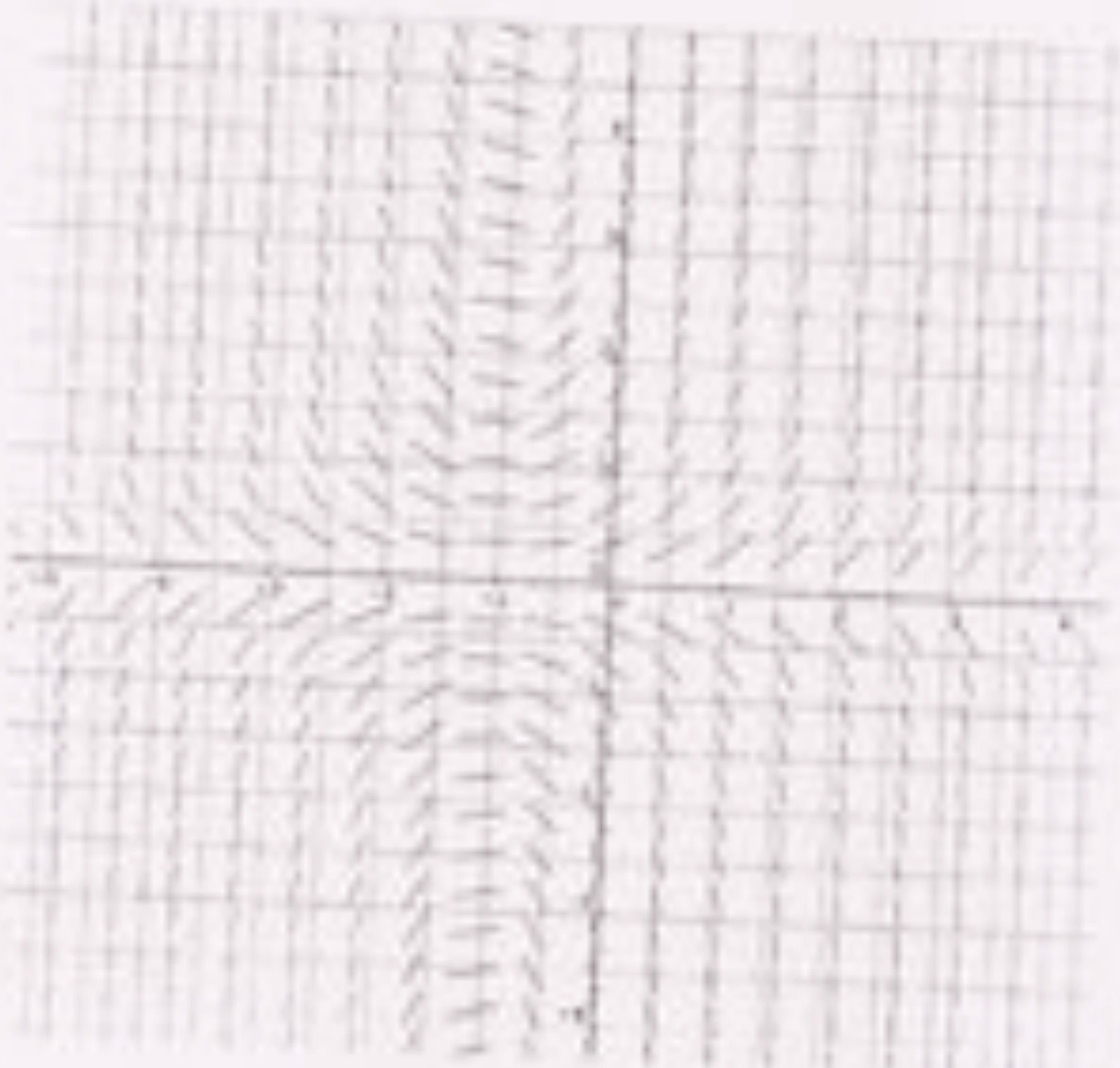
ALL POINTS ON THE CIRCLE $x^2+y^2=1$ ARE SOLUTIONS

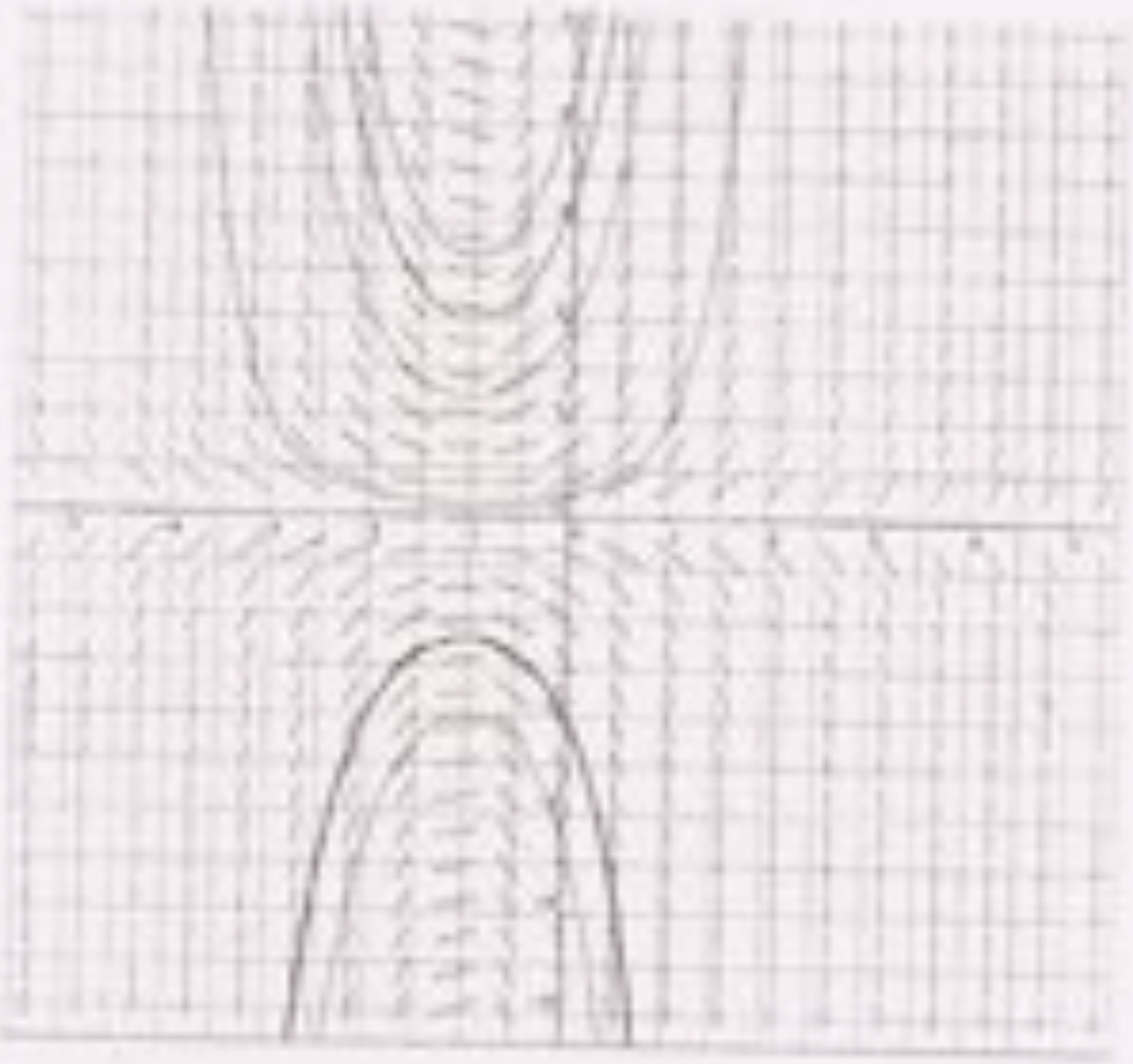
$$\boxed{\left(x,\pm\sqrt{1-x^2}\right)}$$

$\boxed{5}$ CONSIDER THE O.D.E. $\frac{dy}{dt}=(1+t)y$

(a) PLOT THE VECTOR FIELD

(b) AND THE SOLUTION CURVES PASSING THROUGH THE POINTS (-1,2),(-1,-2),(-.5,3),(-1,0),(-.5,-3),(-2,2),(-2,-2),(1,1),(-1,1)





MATH 235 - SPRING 2008

HOMEWORK 1

(c) VERIFY THAT $y(t) = Ce^{(\frac{1}{2}t^2+t)}$ SATISFIES THE O.D.E. REGARDLESS OF THE CHOICE OF C

$$y(t) = Ce^{(\frac{1}{2}t^2+t)}$$

$$\frac{dy}{dt} = Ce^{(\frac{1}{2}t^2+t)}(t+1)$$

SUBSTITUTING $y(t)$ AND $\frac{dy}{dt}$ INTO THE O.D.E., WE GET

$$\frac{dy}{dt} = (1+t)y$$

$$\Rightarrow Ce^{(\frac{1}{2}t^2+t)}(1+t) = (1+t)Ce^{(\frac{1}{2}t^2+t)}$$