Reading Today: 9.4
Tomorrow: 9.5

Today: absorption / dispersion
metals, semiconductors, materials in general.

Up to this point, we showed for dielectric materials that

$$\nabla\cdot\vec{E} = \emptyset \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = \emptyset \qquad \vec{\nabla}\times\vec{B} = \mu\epsilon\frac{\partial\vec{E}}{\partial t}$$

$$\mu = \mu_r\mu_0 \ ; \ \epsilon = \epsilon_r\epsilon_0$$

For conducting materials

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_f}{\epsilon} \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = \emptyset \qquad \vec{\nabla}\times\vec{B} = \mu\vec{J}_f + \mu\epsilon\frac{\partial\vec{E}}{\partial t}$$

Ohm's Law: $\vec{J} = \underset{\substack{\uparrow \\ \text{conductivity}}}{\sigma_c}\vec{E}$ $\qquad \sigma \equiv \frac{1}{\rho}$ ← resistivity

For $\rho_f, \vec{J}_f \rightarrow$

$$\vec{\nabla}\cdot\vec{J}_f = -\frac{\partial\rho_f}{\partial t}$$

$$\vec{\nabla}\cdot(\sigma_c\vec{E}) = -\frac{\partial\rho_f}{\partial t}$$

$$\sigma_c\nabla\cdot\vec{E} = -\frac{\partial\rho_f}{\partial t} = \sigma_c\frac{\rho_f}{\epsilon}$$

$$\int -\frac{\partial\rho_f}{\rho_f} = \int\frac{\sigma_c}{\epsilon}\,dt$$

$$-\ln(\rho_f) + \ln(\rho_f(0))$$

$$-\ln\left(\frac{\rho_f(t)}{\rho_f(0)}\right) = \frac{\sigma_c}{\epsilon}t$$

$$\rho_f(t) = \rho_f(0)\,e^{-\frac{\sigma_c}{\epsilon}t}$$

ρ_f we set to $\emptyset$.

$$\vec{J}_f = \sigma_c \vec{E}$$

$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu \vec{J} + \mu\epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\times\left[\vec{\nabla}\times\vec{B} = \mu\sigma_c \vec{E} + \mu\epsilon \frac{\partial \vec{E}}{\partial t}\right]$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu\sigma_c \vec{\nabla}\times\vec{E} + \mu\epsilon \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = \mu\sigma_c\left(-\frac{\partial \vec{B}}{\partial t}\right) + \mu\epsilon \frac{\partial}{\partial t}\left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\rightarrow \quad \nabla^2\vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} + \mu\sigma_c \frac{\partial \vec{B}}{\partial t}$$

The solution to this is

$$\tilde{A} e^{i\tilde{k}x - i\omega t} \; ; \; \tilde{k} = k + iK$$

Show that this a solution and solve for k an K.

$$\nabla^2\vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma_c \frac{\partial \vec{E}}{\partial t}$$

Answer: $\tilde{k} = \sqrt{\mu\epsilon\omega^2 + i\mu\sigma_c\omega} = k + iK$

$$(k+iK)^2 = \mu\epsilon\omega^2 + i\mu\sigma_c\omega$$

$$(k^2 - K^2) + 2ikK = \mu\epsilon\omega^2 + i\mu\sigma_c\omega$$

$$k^2 - K^2 = \mu\epsilon\omega^2 \; ; \; 2kK = \mu\sigma_c\omega$$

a) Show that this solution also solves $\nabla^2\vec{E} = \mu\tilde{\epsilon}\frac{\partial^2 \vec{E}}{\partial t^2}$; solve for $\tilde{\epsilon}(\mu, \epsilon, \omega, \sigma)$

b) show $\vec{\nabla}\times\vec{B} = \mu\tilde{\epsilon}\frac{\partial \vec{E}}{\partial t}$ for harmonic waves

$$\tilde{\epsilon} = \epsilon + i\sigma_c/\omega = \tilde{\epsilon}_r \epsilon_0$$

$$\tilde{\epsilon}_r = \epsilon_r + \frac{i\sigma_c}{\omega\epsilon_0}$$

$$\mu\sigma_c\vec{E} + \mu\epsilon\frac{\partial \vec{E}}{\partial t}$$

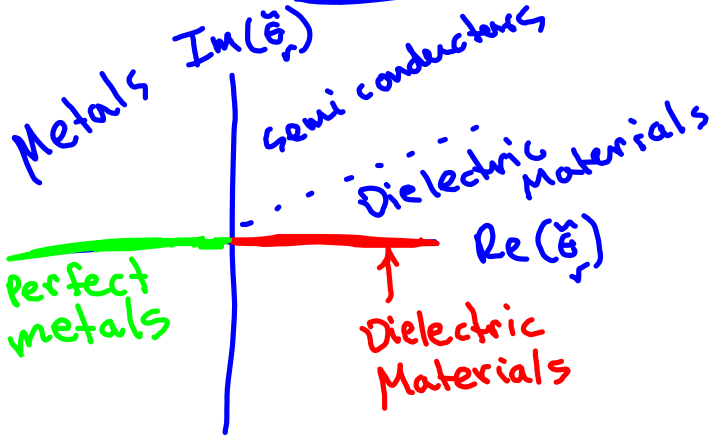
$$\frac{\mu\sigma_c}{-i\omega}\frac{\partial \vec{E}}{\partial t} + \mu\epsilon\frac{\partial \vec{E}}{\partial t}$$

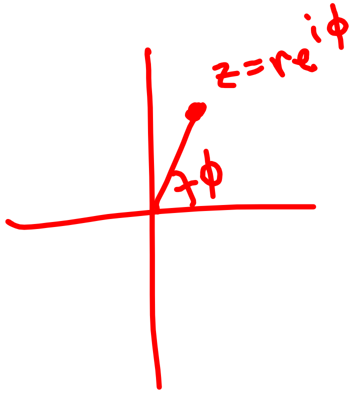
$$\left(\mu\epsilon + \frac{i\mu\sigma_c}{\omega}\right)\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{E} = \emptyset \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

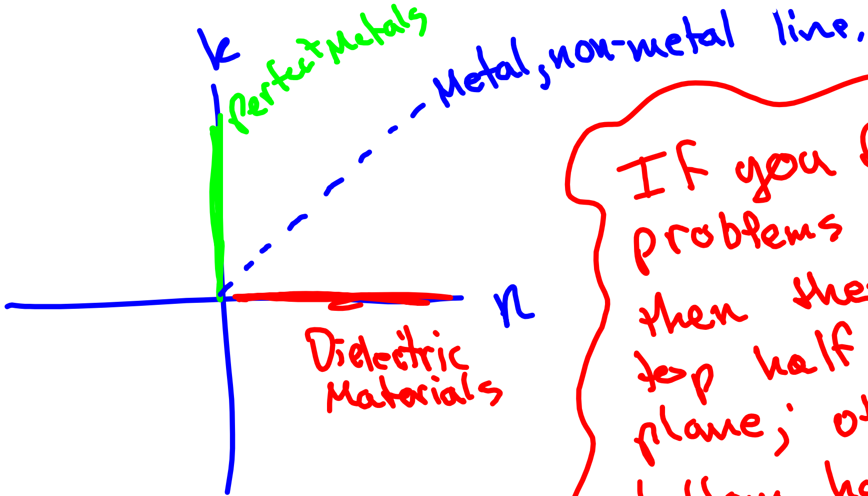
$$\vec{\nabla}\cdot\vec{B} = \emptyset \qquad \vec{\nabla}\times\vec{B} = \mu\tilde{\epsilon}\frac{\partial \vec{E}}{\partial t}$$

The complex dielectric constant





Rather than $\tilde{\epsilon}_r$, you will see literature quote $\sqrt{\tilde{\epsilon}_r} = \tilde{n} = n + ik$



If you formulate your problems as $f(\vec{r})e^{-i\omega t}$ then these are in the top half of the complex plane; otherwise the bottom half.

Let's say we have an absorbing material w/ normally incident light on it

Let's say it's very thick, $T = \emptyset$.

How much energy is lost $= 1 - R$

$$R = \frac{|1-\breve{\beta}|^2}{|1+\breve{\beta}|^2} = \frac{(1-\breve{\beta})(1-\breve{\beta}^*)}{(1+\breve{\beta})(1+\breve{\beta}^*)}$$

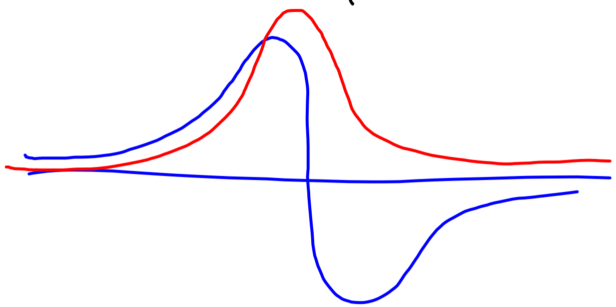
$$\breve{\beta} = \sqrt{\frac{\mu_1 \breve{\epsilon}_2}{\mu_2 \breve{\epsilon}_1}} \qquad \breve{\epsilon}_1 = \epsilon_1 \qquad \mu_1 = \mu_2 = \mu_0$$

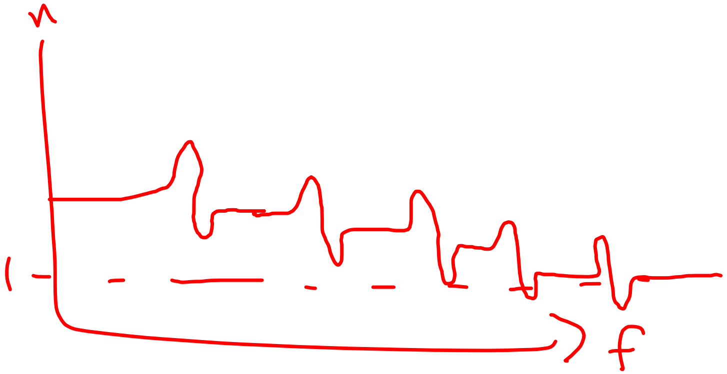
$$\breve{\beta} = \sqrt{\frac{\breve{\epsilon}_2}{\epsilon_1}} \quad ; \quad \breve{\beta} + \breve{\beta}^* = 2\,\mathrm{Re}[\breve{\beta}]$$

$$R = \frac{1 - \breve{\beta} - \breve{\beta}^* + \breve{\beta}\breve{\beta}^*}{1 + \breve{\beta} + \breve{\beta}^* + \breve{\beta}\breve{\beta}^*} = \frac{1 - 2\,\mathrm{Re}[\breve{\beta}] + |\breve{\beta}|^2}{1 + 2\,\mathrm{Re}[\breve{\beta}] + |\breve{\beta}|^2}$$

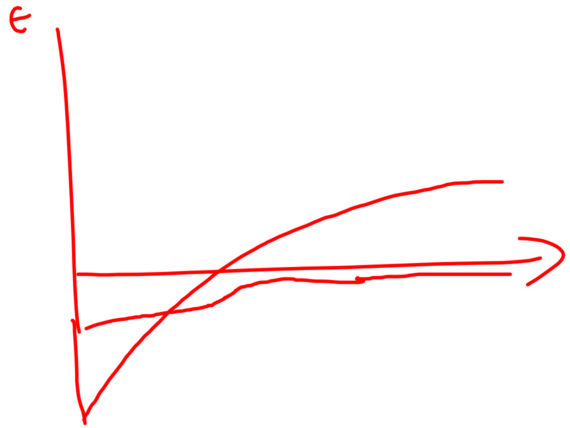
$$R = 1 - \frac{4\,\mathrm{Re}[\breve{\beta}]}{1 + 2\,\mathrm{Re}[\breve{\beta}] + |\breve{\beta}|^2}$$

$$\text{Loss} = \frac{4\,\mathrm{Re}[\breve{\beta}]}{1 + 2\,\mathrm{Re}[\breve{\beta}] + |\breve{\beta}|^2} \qquad \breve{\beta} = \sqrt{\frac{\breve{\epsilon}_2}{\epsilon_1}}$$




n
1
f


absorption
f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