Note Title 10/20/2006

10/20/06

Digression: Amplitude Modulation

consider the sum of two sinusoids of almost the same frequency

$\sin((\omega-\epsilon)t) + \sin((\omega+\epsilon)t)$

where $\epsilon \ll \omega$

show that this can be written

$$2\underbrace{\cos(\epsilon t)}_{\text{modul. freq}}\underbrace{\sin(\omega t)}_{\text{carrier freq.}}$$

cf. wikipedia on Amplitude modulation

Fourier Series

Let $f(x)$ be periodic on $[-\ell, \ell]$

any finite interval is OK

Then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x/\ell) + b_n \sin(n\pi x/\ell)$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/\ell}$$

a_i, b_i real
c_i complex

$$c_k = \frac{1}{2}(a_k - ib_k) \quad c_{-k} = \frac{1}{2}(a_k + ib_k)$$

Think of the F.S. as being like a vector super position

$$\vec{q} = q_1 \hat{e}_1 + q_2 \hat{e}_2 + q_3 \hat{e}_3$$
$$= \sum_{i=1}^{3} q_i \hat{e}_i$$

expansion coefficients

basis vectors

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/l}$$

exp. coef

basis fcns.

$$(\vec{x}, \vec{y}) = \sum^{N} x_i y_i \qquad \vec{x}, \vec{y} \in R^N$$

$i=1$

$$(u,v) = \int_{-\ell}^{\ell} u(x)\,v(x)\,dx$$

$$\varphi_k(x) \equiv \sin(k\pi x/\ell)$$

$$\psi_k(x) \equiv \cos(k\pi x/\ell)$$

$$\boxed{\begin{gathered}(\varphi_i,\varphi_j) = (\psi_i,\psi_j) = \ell\,\delta_{ij} \\ (\varphi_i,\psi_j) = 0\end{gathered}}$$

$$e^{i\omega t P}$$

$$e^{ix} = 1 + x + \frac{x^2}{2} + \cdots$$

$$e^{iP} = 1 + P + \frac{P^2}{2} + \cdots$$

$$P^2 = I$$

$$P^{2N} = I \qquad P^{2N+1} = P$$

$$e^{iP} = \underset{I}{1} + \underset{I}{\frac{P^2}{2}} + \underset{I}{\frac{P^4}{4}} + \underset{P}{P} + \underset{P}{\frac{P^3}{3}}$$

$$(\quad \cos \quad) I + i(\ \sin\ ) P$$