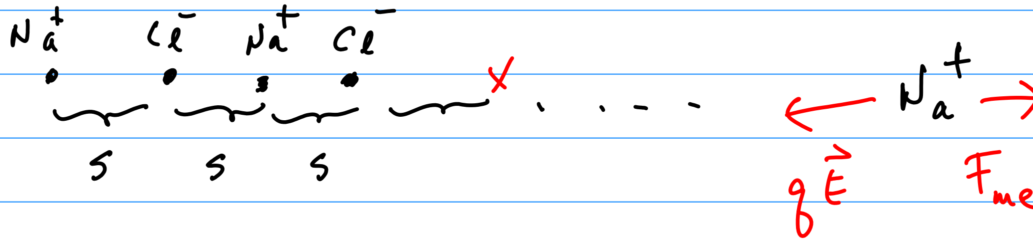
Lecture 14

Shadowitz: 4-2 capacitance, 4-3 electrostatic energy

1-D salt crystal



Questions?

How do I calculate the work required to bring in another atom (congruous)?

Work-Energy theorem

$$W_{net} = \int \vec{F}_{net} \cdot d\vec{r} = \Delta KE$$

non cons   cons

$$W_{nc} + W_{con} = \Delta KE$$

$$W_{con} = -\Delta PE$$

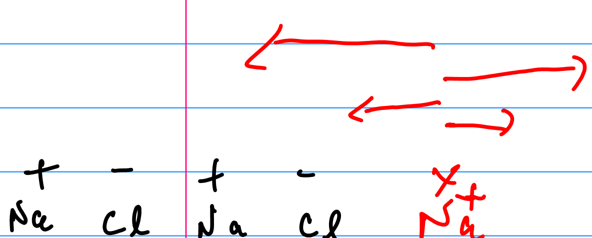
$$W_{nc} = \Delta(KE + PE)$$

0 at ∞

$$W_{me} = \Delta PE = q\Delta V = q(V_f - \cancel{V_i})$$

Questions:

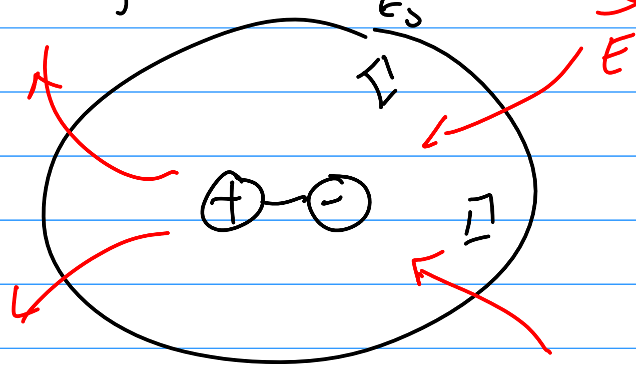
E goes through other charges and does not cancel since charges are at different distances



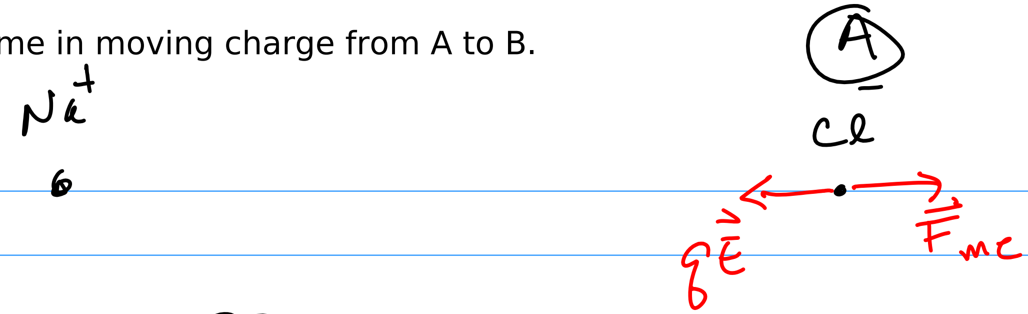
Qnet = 0 doesn't mean E = 0 everywhere.

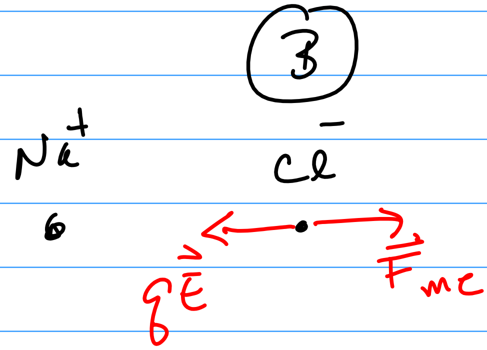
Flux in = minus flux out so net flux = 0

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0} = 0$$



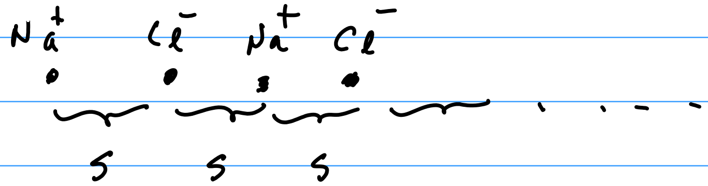
Work done by me in moving charge from A to B.





$$W_{me} = -\int_{A}^{B} q\vec{E}\cdot d\vec{r} = q\Big(\underbrace{-\int \vec{E}\cdot d\vec{r}}_{\Delta V}\Big) = q\big(V_B - \underset{0\,@\,\infty}{V_A}\big)$$

$$W_{me} = \Delta PE = \frac{q(-q)}{4\pi\epsilon_0 s}$$ 1 charge present



$$W_{me} = qV_{total} = q\left[\frac{-q}{4\pi\epsilon_0 s} + \frac{q}{4\pi\epsilon_0 2s} - \frac{q}{4\pi\epsilon_0 3s} \ldots\right]$$

$$= \frac{-q^2}{4\pi\epsilon_0 s}\left[1 - \frac{1}{2} + \frac{1}{3} + \ldots\right] = -\frac{.693\, q^2}{4\pi\epsilon_0 s}$$

x-ray data yields s.

Questions:

How do I calculate work done by me to assemble all the charges (congruous)?

Will the charges come together or do I have do work to assemble them?

Let the next charge be at infinity. Let it go.

$$0 = W_{me} = \Delta(KE + PE) : \underbrace{(KE + PE)_f}_{\text{negative}} - (\underbrace{KE}_{0} + \underbrace{PE}_{0})_i = 0$$

$W_{me} = W_{\text{non conservative}}$

$\Downarrow$

$KE_{final} = PE_f$ which is positive

so charge moves in on its own.

How much work do I have to do to assemble the crystal (congrous)?

$$W_1 = 0 \qquad W_2 = \frac{-q^2}{4\pi\epsilon_0}\frac{1}{s} \qquad W_3 = \frac{-q^2}{4\pi\epsilon_0}\left[\frac{1}{s} - \frac{1}{2s}\right]$$

$$W_4 = \frac{-q^2}{4\pi\epsilon_0}\left[\frac{1}{s} - \frac{1}{2s} + \frac{1}{3s}\right]$$

Questions

How do I calculate the work required to assemble a continuous charge distribution (congrous)?

$$dW_{me} = V\,dq$$

Voltage at the point where dq is placed due to the charges already present but NOT the charges yet to be brought in.

Questions:

How do you calculate this in a simple example (congrous)?

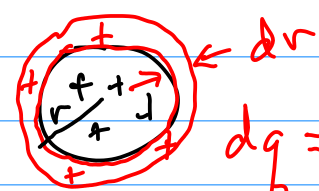
Lines and planes of charges have problems with infinities.
Let's calculate the energy needed to assemble a sphere of charge instead.

How much work do I have to do to assemble a uniformly charged sphere?

To bring in the first dq requires no work since V present is zero.

What do I bring in next (congruous)?

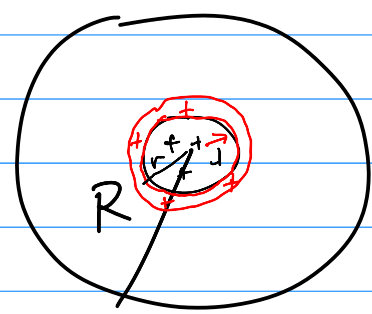
ρ $\frac{\text{Coul}}{m^3}$ is charge density



$$dq = \rho\, 4\pi r^2\, dr$$

$$W = \int V\,dq$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \rightarrow \frac{1}{4\pi\epsilon_0}\frac{\rho\frac{4}{3}\pi r^3}{r}$$



$$W = \int_0^R \frac{1}{4\pi\epsilon_0}\rho^2\frac{(4\pi)^2}{3} r^4\,dr = \frac{4}{15}\frac{\pi\rho^2 R^5}{\epsilon_0}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{q^2}{R}$$

Questions

For an electron R is zero and W is infinite. As far as we know the electron is a point charge. Our model says it has infinite energy but we can not extract that energy. It is just something nature has given us but we can't manipulate it.

Main result from above is

$$\boxed{W = \int V \, dq}$$

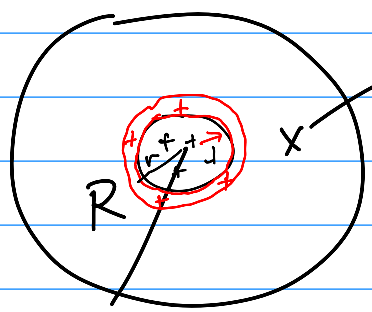
Analagous questions about electricity and magnetism?

How is electricity and magnetism related to gravity (analagous)?

http://en.wikipedia.org/wiki/Gravitoelectromagnetism

How do I calculate the work if it is easy to determine the voltage at every point in the charge distribution due to all the charges that are present?

This voltage is NOT the voltage due to only the charges that have been brought it!!!



$$V = -\int_{\infty}^{r} \vec{E}_{tot} \cdot d\vec{r}$$

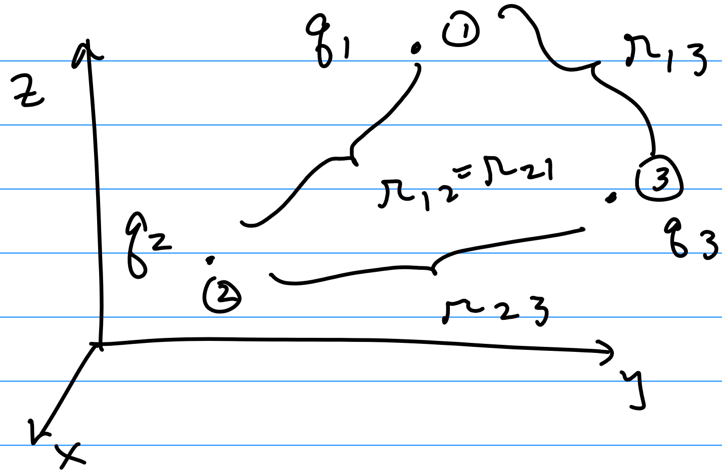
Questions

How do I calculate the work required given the voltage at each point in the charge distribution due to all charges present (congruous)?

What simple example can I use to understand how this is done (modifying)?

For Wednesday's lecture below

Use a three charges to find work (two is too simple and four too complicated).



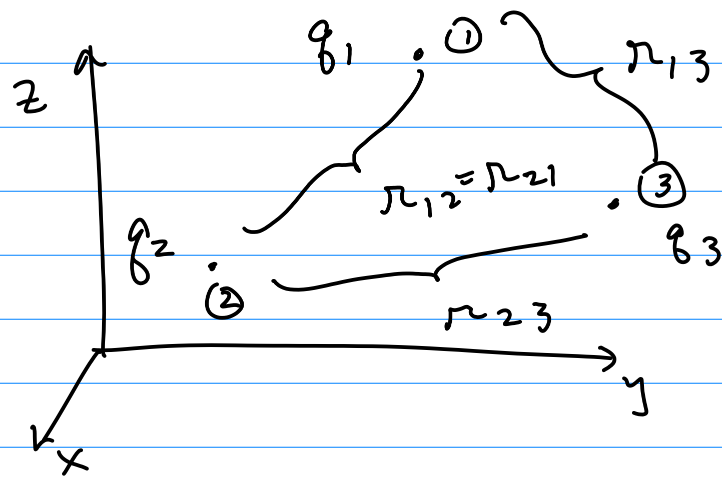
Let charge 1 be brought in first, then charge 2, then charge 3.

$$W_{me} = \frac{1}{4\pi\epsilon_0}\left(\right.$$

$\frac{q_1 q_2}{r_{12}}$

$\frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}$

What is the voltage at each charge due to all others (informational)?



$$V_{①} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}}\right)$$

$$V_{②} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{12}} + \frac{q_3}{r_{23}}\right)$$

$$V_{③} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

$$W_{me} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$2W_{me} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left[\underbrace{q_1\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}}\right)}_{V(1)} + \underbrace{q_2\left(\frac{q_3}{r_{23}} + \frac{q_1}{r_{12}}\right)}_{V(2)} + \underbrace{q_3\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)}_{V(3)}\right]$$

$$W_{me} = \frac{1}{2}\sum_{i=1}^{N} q_i \, V(P_i)$$

i^{th} point

Questions