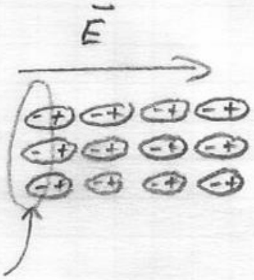
Fields in matter - A (short) review of $\bar{D}$ + $\bar{H}$ (in statics)

We start E+M by writing down Gauss's law, $\nabla \cdot \bar{E} = \rho/\varepsilon_0$
Then we do a bunch of problems in vacuum + in conductors.
How does E+M change when we move to dielectrics?

Well, Gauss's law remains true always: $\nabla \cdot \bar{E} = \rho/\varepsilon_0$
Charges make diverging $\bar{E}$-fields.

But when you're in a polarizable dielectric, you have to be a bit more careful with your source term ρ.

If you apply an $\bar{E}$ field to a dielectric, the atoms polarize:



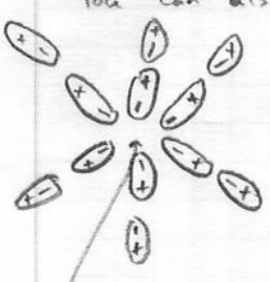
bound negative charge

A net neutral material remains overall neutral, but the + and - charges shift slightly, leading to an accumulation of + and - charges on the edges. We call these charge accumulations <u>bound</u> <u>charges</u>: charges that show up only because some material has become polarized.

Back in intermediate E+M, you may have discovered that these <u>surface</u> bound charges can be described in terms of the polarization $\vec{P}$ via:

$\sigma_b = \hat{n} \cdot \vec{P}$ where $\hat{n}$ is the surface normal vector.

You can also get <u>volume</u> bound charges if your polarization has divergence:



accumulation of negatives at the center - bound volume charge

These bound charges are real charges that will absolutely make fields according to Gauss's law, which need not change if we account for all the charges:

$$\nabla \cdot \bar{E} = \rho_{total}/\varepsilon_0$$

We often find it convenient to separate ρ into free and bound terms:

$\rho_{total} = \rho_{free} + \rho_{bound}$ Where ρ_{free} is some net charge that you put in the system that'd be there whether the material is polarized or not.

* Super-important note: In this context, the words free + bound have nothing to do with whether those charges are <u>free to move</u>.

Now, since ρ_b depends on $\vec{P}$ which depends on $\vec{E}$, in matter Gauss's law looks like

$$\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0}\left(\rho_b(\vec{E}) + \rho_f\right)$$

Gauss's law is often used as a recipe for finding $\vec{E}$ given ρ. If you need to know $\vec{E}$ to find all the ρ's, you're gonna have a bad time.

Clever solution: If $\rho_b = -\nabla \cdot \vec{P}$, then

$$\nabla \cdot \vec{E} = -\frac{1}{\varepsilon_0}\nabla \cdot \vec{P} + \rho_f/\varepsilon_0$$

$$\Rightarrow \nabla \cdot \left(\vec{E} + \frac{1}{\varepsilon_0}\vec{P}\right) = \frac{1}{\varepsilon_0}\rho_f$$

$$\Rightarrow \nabla \cdot \left(\varepsilon_0 \vec{E} + \vec{P}\right) = \rho_f$$

So if we define some auxiliary field $\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}$, we can write

$$\boxed{\nabla \cdot \vec{D} = \rho_f}$$

which lets us solve problems more easily in polarizable media, since the right hand side is independent of the field we're solving for.

The "real" field is still $\vec{E}$, since that's the field produced by all the charges that actually exist (ρ_f and ρ_b). $\vec{D}$ is the thing you get if you abstract out the hard-to-deal-with sources.

To know $\vec{D}$ we need to know its div + curl (remember Helmholtz):

$$\nabla \times \vec{D} = \nabla \times \left(\varepsilon_0 \vec{E} + \vec{P}\right)$$

If we're in statics (so $\nabla \times \vec{E} = 0$) and the polarization is also curl-less (not <u>always</u> true), then:

$$\boxed{\nabla \times \vec{D} = 0}$$

And from these, boundary conditions follow immediately:

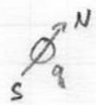
$$\boxed{\begin{aligned} D_{1,\perp} - D_{2,\perp} &= \sigma_f \\ D_{1,\parallel} - D_{2,\parallel} &= 0 \end{aligned}}$$

(clicker on curly $\vec{P}$)

So what about $\vec{B}$ in matter? We start with

$\nabla \times \vec{B} = \mu_0 \vec{J}$ Where $\vec{J}$ is some current – some movement of electrons. Henceforth we'll refer to this regular old transport-of-electrons current as free current – $\vec{J}_f$

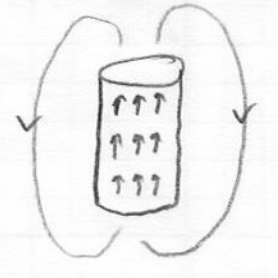
There's another source of magnetism, though: the magnetism that comes from the intrinsic magnetic moments of elementary particles.



Charged particles have intrinsic magnetic moments that exist for reasons that apparently involve relativistic quantum mechanics. It's a bit over my head. But one thing I can say for sure is that it's not because an electron is literally a spinning ball of charge, making a current.

Similarly, atoms have a magnetism in addition to that of their component particles on account of interactions between the electron and nucleus. This orbital magnetic moment has no good classical analog, since electrons do not literally orbit nuclei (which would also make a current).

Anyway, atoms generally have magnetic dipole moments. Collectively these can lead to macroscopic magnetic fields that can be modeled as coming from macroscopic <u>bound currents</u>, which aren't really currents in the usual sense



cylinder full of dipoles makes the same field as

a cylinder with a surface current wrapped around the outside

You (hopefully) derived that $\vec{K}_{bound} = \vec{M} \times \hat{n}$

and $\vec{J}_{bound} = \nabla \times \vec{M}$

To do E&M in matter, we can add these sources into Ampere's law:

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \vec{J}_b \right)$$

But when we go to solve for $\vec{B}$, we run into the same problem as before: $\vec{J}_b$ depends on $\vec{B}$. And we pull the same trick:

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \vec{J}_b \right) \qquad \vec{J}_b = \nabla \times \vec{M}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}_f + \nabla \times \mu_0 \vec{M}$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f \qquad \text{Define } \vec{H} \equiv \vec{B}/\mu_0 - \vec{M}$$

$$\Rightarrow \boxed{\nabla \times \vec{H} = \vec{J}_f}$$

which is Ampere's law in terms of free sources only. $\vec{H}$ is no more fundamental than $\vec{D}$, but it is frequently useful; so frequently that some people use it quite a bit more than $\vec{B}$.

If $\vec{M}$ has no divergence, then $\boxed{\nabla \cdot \vec{H} = 0}$ and boundary conditions on $\vec{H}$ pop out:

$$\boxed{\bar{H}_{1,\perp} - \bar{H}_{2,\perp} = 0 \qquad \bar{H}_{1,\parallel} - \bar{H}_{2,\parallel} = 0}$$