PHGN 462 Homework 8

1) (From Pollack and Stump 13.12)

This next one lets us derive the equations for group and phase velocity. We're going to start with a Gaussian wave pulse – that is, an electromagnetic wave with an overall Gaussian profile and a relatively short length. Underneath that Gaussian profile we still have the usual sinusoidal ripples at some frequency ω. Such a wave is made up of a whole bunch of different frequencies, centered on ω. We'll represent the time-domain wave amplitude as the Fourier transform of the frequency-domain wave:

$$\phi(x,t) = \int_{-\infty}^{\infty} e^{i(kx-\omega t)} f(k) \frac{dk}{2\pi}$$

where $f(k) = f_0 e^{-(k-k_0)^2 a^2}$ is a function describing the contribution to the pulse from any particular wavenumber *k* (which corresponds to some particular frequency). Recall that the Fourier transform of a Gaussian is a Gaussian – if it's a Gaussian pulse in the time domain, it's also a Gaussian in the frequency domain.

The Fourier transform integrates *k* from $(-\infty, \infty)$, but the integrand is peaked at $k = k_0$, and the width of the peak is of order $1/a$. Within a sufficiently narrow span, ω(k) will be locally linear, and can be approximated by:

$$\omega(k) = \omega(k_0) + (k - k_0)\omega'(k_0)$$

a) Evaluate the integral, and obtain explicitly the function $\phi(x,t)$. (Hint: Let $k = k_0 + q$, change the variable of integration to q, and evaluate the integral with tables or Mathematica.)

b) Show explicitly that the phase velocity is $\omega(k_0)/k_0$ and the group velocity is $\omega'(k_0)$. Keep in mind that you can find a velocity by tracking a particular value of the argument of a function.

2) We're going to be spending a lot of time talking about how E&M waves propagate in various waveguides, and not a lot of time thinking about why we bother to propagate them. Root around in books or on the interwebs and learn about the actual applications of microwave waveguides. Write up a short description of two different applications. It would be nice if at least one of them came into common usage after 1960 (a lot of microwave tech is pretty old).

And don't take advantage of this problem by directly copying and pasting stuff from, say, the first and second websites that come up when you search for "microwave waveguides." I know which sites those are. I also know what the third and fourth hits are, before you get any clever ideas.