NLO with guided waves.

1) solve linear eqn for transverse mode structure.

$$\nabla_T^2 A + (k_0^2 n^2(r) - k_z^2) A = 0$$ assume azimuthal symmetry.

modal solutions $u_m(r)$

$$A(r,z,t) = \sum_m a_m u_m(r) e^{i(k_{zm} z - \omega t)}$$

- each mode has its own propagation constant.
- if all light is guided → u_m is a complete set
- otherwise there are unguided waves too (radiation modes
- a_m = amplitude coeff. Constant if linear, no loss.

2) calculate NL polarization e.g. $\chi^{(3)}$ THG

$$P^{NL} \propto \chi^{(3)} A^3$$ picking out $\omega_3 = 3\omega_1$

easiest if input is all same mode

$$P^{NL} \sim a_{1m}^3(z)\, u_{1m}^3(r)\, e^{i3k_{1m}z}$$

3) project P^{NL} onto guided modes for $\omega_3 = 3\omega_1$

separate eqns for $a_{3m}(z)$

$$2ik_{3m'} \frac{da_{3m'}}{dz} e^{ik_{3m'}z} \propto k_{1m} \chi^{(3)} a_{1m}^3 e^{i3k_{1m}z} \cdot \underbrace{2\pi \int_0^\infty u_{3m'}^*(r)\, u_{1m}^3(r)\, r\,dr}_{\text{mode overlap}}$$

→ lower effective $\chi^{(3)}$

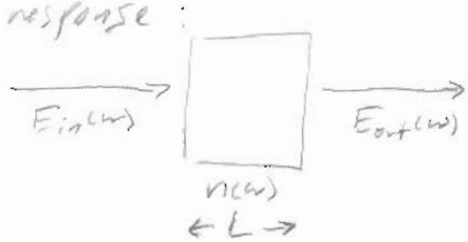
note phase matching is mode-selective.

$$\Delta k = 3k_{1m} - k_{3m'}$$

can use modes to assist phase matching.

Time vs. frequency domain

Linear response:



$$E_{out}(\omega) = E_{in}(\omega)\, e^{ik_0 n(\omega) L}$$

linear propagation is easiest to describe in freq. space.

- linear systems: $H(\omega)$ = transfer fcn, freq. response

$$F_{out}(\omega) = H(\omega) F_{in}(\omega)$$

$$f_{out}(t) = \mathfrak{F}^{-1}\{H(\omega) F_{in}(\omega)\}$$

$$= h(t) \otimes f_{in}(t) \qquad \text{convolution theorem}$$

where $$= \int_{-\infty}^{\infty} h(\tau) f_{in}(t-\tau)\, d\tau \qquad \text{is convolution.}$$

if input is $f_{in}(t) = \delta(t)$ (impulse)

$$f_{out}(t) = \int_{-\infty}^{\infty} h(\tau)\, \delta(t-\tau)\, d\tau = h(t)$$

$$\therefore\ h(t) = \mathfrak{F}^{-1}\{H(\omega)\} \quad \text{is impulse response.}$$

At a microscopic level, $p(t)$ is the induced dipole when $E(t)$ is in and $P(t) = N_a p(t)$ is the collective response.

When we solve for $\chi^{(1)} = N_a P/E$, our method gives $\chi^{(1)}(\omega)$

recall steps: $E(t) = E_0 e^{-i\omega t} + c.c.$

$$x^{(1)}(t) = x_0^{(1)} e^{-i\omega t} + c.c.$$

$$x_0^{(1)} = -(e/m) E_0 / D(\omega)$$

$$\chi^{(1)}(\omega) = N_a e^2/m \,/\, D(\omega)$$

So what is the impulse response? For a range of input freq:

$$P^{(1)}(\omega) = \chi^{(1)}(\omega) E(\omega)$$

Alternative method: take FT of 2^{nd} order eqn.

note that $\mathfrak{F}\{\partial f/\partial t\} = -i\omega F(\omega)$

$\rightarrow$ eqn for $\chi^{(1)}(\omega)$ with $E(\omega)$ driving

Now in time domain,

$$P^{(1)}(t) = \mathfrak{F}^{-1}\{\chi^{(1)}(\omega) E(t)\}$$

Recognize $\chi^{(1)}(\omega)$ as a transfer function.

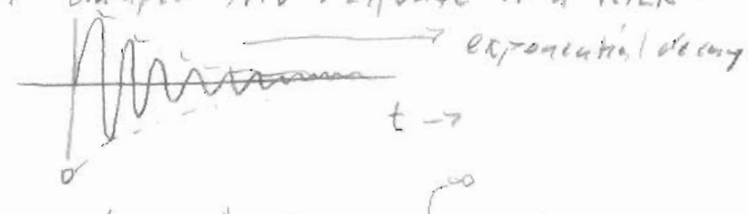
let $R^{(1)}(t) \equiv 2\pi \mathfrak{F}^{-1}\{\chi^{(1)}(\omega)\} \equiv$ impulse response

Then $P^{(1)}(t) = \varepsilon_0 \int_{-\infty}^{\infty} R^{(1)}(\tau) E(t-\tau)\, d\tau = R^{(1)} \otimes E$

(normally have $\frac{1}{2\pi}$ in front. this is absorbed into defn of R)

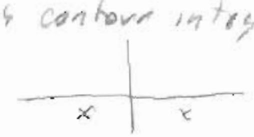
What is the nature of $R^{(1)}(t)$?

- causality requires $R^{(1)}(t) = 0$ for $t < 0$
- expect? damped SHO response to a kick:



- Proof: 1) $\mathfrak{F}^{-1}\{\chi^{(1)}(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{N(e^2/m)}{\omega_0^2 - \omega^2 - 2i\omega\gamma} e^{-i\omega t}\, d\omega$

requires contour integration: poles are off real axis:



for $t < 0$ close on upper half $\rightarrow 0$

for $t > 0$ close lower.