

$$\begin{aligned}
 \mathbf{A} &= 7 \cdot \left[ \frac{40}{49} \cdot \frac{-2}{\sqrt{40}} \cdot \frac{6}{\sqrt{40}} + \left( \frac{\sqrt{40}}{7} \cdot \frac{-2}{\sqrt{40}} + \frac{3}{7} \right) \cdot \frac{\sqrt{40}}{7} \cdot \frac{6}{\sqrt{40}} \right] \mathbf{a}_r \\
 &+ 7 \cdot \left[ \frac{\sqrt{40}}{7} \cdot \frac{3}{7} \cdot \frac{6}{\sqrt{40}} \cdot \frac{-2}{\sqrt{40}} + \left( \frac{\sqrt{40}}{7} \cdot \frac{-2}{\sqrt{40}} + \frac{3}{7} \right) \cdot \frac{3}{7} \cdot \frac{6}{\sqrt{40}} \right] \mathbf{a}_\theta \\
 &+ 7 \cdot \left[ \frac{-\sqrt{40}}{7} \cdot \frac{36}{40} + \left( \frac{\sqrt{40}}{7} \cdot \frac{-2}{\sqrt{40}} + \frac{3}{7} \right) \cdot \frac{-2}{\sqrt{40}} \right] \mathbf{a}_\phi \\
 &= \frac{-6}{7} \mathbf{a}_r - \frac{18}{7\sqrt{40}} \mathbf{a}_\theta - \frac{38}{\sqrt{40}} \mathbf{a}_\phi \\
 &= -0.8571\mathbf{a}_r - 0.4066\mathbf{a}_\theta - 6.008\mathbf{a}_\phi
 \end{aligned}$$

Note that  $|\mathbf{A}|$  is the same in the three systems; that is,

$$|\mathbf{A}(x, y, z)| = |\mathbf{A}(\rho, \phi, z)| = |\mathbf{A}(r, \theta, \phi)| = 6.083$$

### PRACTICE EXERCISE 2.1

- (a) Convert points  $P(1, 3, 5)$ ,  $T(0, -4, 3)$ , and  $S(-3, -4, -10)$  from Cartesian to cylindrical and spherical coordinates.
- (b) Transform vector

$$\mathbf{Q} = \frac{\sqrt{x^2 + y^2} \mathbf{a}_r}{\sqrt{x^2 + y^2 + z^2}} - \frac{yz \mathbf{a}_z}{\sqrt{x^2 + y^2 + z^2}}$$

to cylindrical and spherical coordinates.

- (c) Evaluate  $\mathbf{Q}$  at  $T$  in the three coordinate systems.

### ANSWER

- (a)  $P(3.162, 71.56^\circ, 5)$ ,  $P(5.916, 32.31^\circ, 71.56^\circ)$ ,  $T(4, 270^\circ, 3)$ ,  
 $T(5, 53.13^\circ, 270^\circ)$ ,  $S(5, 233.1^\circ, -10)$ ,  $S(11.18, 153.43^\circ, 233.1^\circ)$

- (b)  $\frac{\rho}{\sqrt{\rho^2 + z^2}} (\cos \phi \mathbf{a}_\rho - \sin \phi \mathbf{a}_\phi - z \sin \phi \mathbf{a}_z)$ ,  $\sin \theta (\sin \theta \cos \phi - r \cos^2 \theta \sin \phi) \mathbf{a}_r + \sin \theta \cos \theta (\cos \phi + r \sin \theta \sin \phi) \mathbf{a}_\theta - \sin \theta \sin \phi \mathbf{a}_\phi$
- (c)  $0.8\mathbf{a}_r + 2.4\mathbf{a}_\theta$ ,  $0.8\mathbf{a}_\rho + 2.4\mathbf{a}_z$ ,  $1.44\mathbf{a}_r - 1.92\mathbf{a}_\theta + 0.8\mathbf{a}_\phi$