Reading tomorrow : 9.5

Errata: $\langle f^2 \rangle \neq \tilde{f}\tilde{f}^* = \frac{1}{2}\tilde{f}\tilde{f}^*$

$\langle fg \rangle \neq \tilde{f}\tilde{g}^* = \frac{1}{2}\tilde{f}\tilde{g}^*$

For today : <u>Dispersion.</u>

A material is dispersive when $\epsilon, \mu,$ or σ depends on ω.

Why would the constant depend on ω.

The book goes through and treat the atom like an electron tied to a nucleus by spring.

$$F_{\text{binding}} = -m\omega_0^2 \underset{\substack{\uparrow \\ \text{displacement}}}{x}$$

$$F_{\text{damping}} = -m\gamma \frac{dx}{dt}$$

$$F_{\text{driving}} = qE_0 e^{-i\omega t}$$

$$\frac{d^2\tilde{x}}{dt^2} + \gamma \frac{d\tilde{x}}{dt} + \omega_0^2 \tilde{x} = \frac{q}{m} E_0 e^{-i\omega t}$$

$$\Rightarrow \tilde{x} = \frac{q/m}{\omega_0^2 - \omega^2 - i\gamma\omega} E_0 e^{-i\omega t}$$

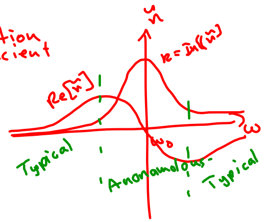
$$\tilde{p} = q\tilde{x} \Rightarrow \tilde{\vec{p}} = \underbrace{\frac{q^2}{m}\left(\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right)}_{\tilde{e} = \epsilon_0 \tilde{\chi}_e} \vec{E}$$

For f electrons in N atoms per volume. Not all electrons have the same ω_0.

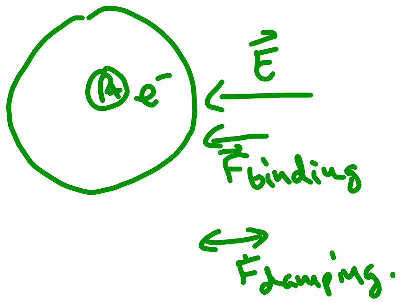
Generally, we make this.

$$\tilde{p} = \underbrace{\frac{q^2 N}{m} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}}_{\tilde{\epsilon}} \tilde{E}$$

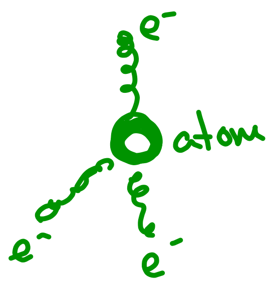
$$\tilde{n} = \sqrt{\tilde{\epsilon}/\epsilon_0} = \underset{\substack{\uparrow \\ \text{index} \\ \text{of refraction}}}{n} + i \underset{\substack{\uparrow \\ \text{extinction} \\ \text{coefficient}}}{k}$$



We assumed we had an isotropic effect.



Good for gases, liquids, amorphous solids.
What about crystals?
If you have an ordered material, then typically the model looks like



springs (bonds in crystal) can have different strengths in different directions.

What's the result of that?

Birefringent materials, where ϵ in one direction is different from another.

In practice, $\tilde{\epsilon}(\lambda)$

1. You model the $\tilde{\epsilon}(f)$ like as a SHM.
2. You experimentally measure $\tilde{\epsilon}(\lambda), \tilde{\epsilon}(f)$ and then apply some fit parameters.

Common Forms

Cauchy Formula

$$\mathrm{Re}(n) = 1 + A\left(1 + \frac{B}{\lambda_0^2}\right)$$

Sellmeier Eqn

Typically the better one. →

$$\epsilon(\lambda_0) = 1 + \frac{B_1\lambda_0^2}{\lambda_0^2 - C_1} + \frac{B_2\lambda_0^2}{\lambda_0^2 - C_2} + \frac{B_3\lambda_0^2}{\lambda_0^2 - C_3}$$

What are the effects of dispersion

Some terminology

v_{phase} = speed of wave fronts $= \frac{\omega}{k}$

$v_{group} \approx$ speed of wave packets $= \frac{\partial \omega}{\partial k}$
$\approx$ speed of energy propagation

$\Rightarrow$ Solve for v_{group} for a plane wave in a material (non-dispersive) of index n.

$$v_{group} = \frac{c}{n}$$

$\Rightarrow$ Solve for v_{group} for a dispersive material where $n = n_0 + n_1 \omega$