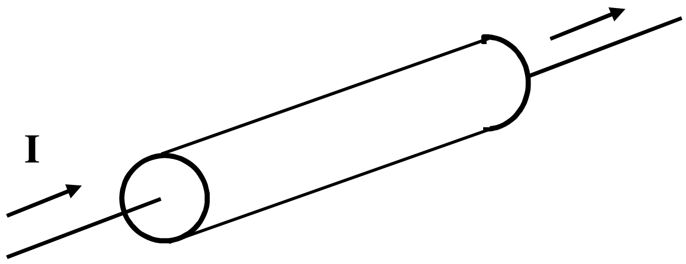
Phys 361 Homework 8

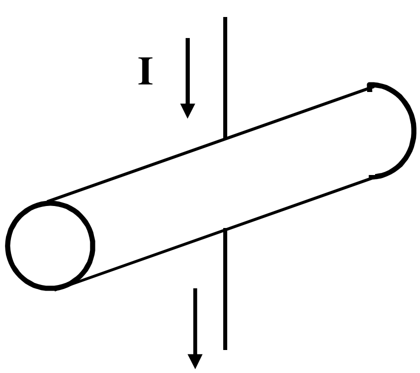
1) Many, perhaps most, commercial resistors are cylindrical in shape, with leads arranged on the long axis of the cylinder:



a) Given a resistor of length L, diameter D, and resistivity ρ, what is the resistance of a commercial resistor when hooked up with the leads in the intended position?

b) Find an actual cylindrical resistor (loot the studio or electronics lab if you don't have one), physically measure L and D as best you can and backtrack out the resistivity of the material that must have been used in its construction. Just for grins, estimate the uncertainties in your L and D measurements, check the uncertainty in the resistance of the resistor, and calculate the resulting uncertainty in your answer for the resistivity (I bet you didn't know this was a lab class).

c) Now let's suppose you were to attach the leads to the top & bottom of the cylinder instead of to the ends (see picture below). Calculate the resistance of this new arrangement (I've always wanted to assign this in Phys 200, but it would break them). You may assume as a good approximation that the current flows strictly vertically and ignore any angular dependence as it fans out. Also plug in your resistivity from (b) and see what you get. Comment on your answer (numeric and algebraic; there's something I found surprising about the algebraic form).



2) (based on Pollack and Stump 7.3)

a) Let's start with a solid sphere of radius a with total charge Q uniformly distributed throughout. The sphere is spinning with some angular velocity $\vec{\omega} = \omega\hat{k}$.

a) Find the current density $\vec{J}(\vec{x})$, in spherical coordinates.

b) Calculate the total current I in this sphere. You might have to think a bit about how to define "current" in this geometry, in particular regarding what cross-sectional area the charges should be flowing through. We will quite likely be using the results here in the future - believe it or not, I don't pick the homework problems completely at random.

3) Okay, so we know that current doesn't drive itself. In a current-carrying wire, there's an electric field pushing the conduction electrons along. But how does that field come about? In electrostatics, fields come from charges. And there can't be any net charge inside the conductor. The battery might be really far away, leaving it an inappropriate source. So when we think real hard we get forced into the conclusion that a current-carrying wire has a *surface* charge distribution σ that is responsible for making the E-field that pushes the current. There's simply no other place that the source of the E could be. Most people find this rather surprising the first time they hear about it.

Consider a coax cable. There is a *solid* (3D) cylindrical wire of radius a carrying a uniform current I in the $\hat{k}$ direction. Surrounding that is a *hollow* (2D) cylindrical sheath of radius b carrying the return current back in the other direction.

Your ultimate goal is to find the surface charge density on the solid interior wire (at $r = a$). You may have to find a variety of other things along the way, like the potential in various regions. To clean things up a little, let's assume that the $z = 0$ end of the cable is grounded (which is to say, the potential on that end is zero).

Some hints:

You can't assume the potential is independent of z in this problem (why not?). But you can assume rotational invariance (potential <u>is</u> independent of φ). In other words, you'll need to find $V(r, z)$ in cylindrical coordinates.

You should probably start from the very beginning ($\nabla^2 V = 0$), assume a product solution, and go from there. As we discussed in class, the separation constant turns out to be zero. You should discuss how it is we can reach that conclusion.

You may be able to guess the form of the electric field that's driving the current, and confirm it (or infer it in the first place) from Ohm's law. That'll give you some leverage regarding the potential for $r < a$. Remember, any real conductor has at least a little resistance, so Ohm's law is relevant.

You'll also need the electric field outside the entire thing (for $r > b$). You might reasonably guess $E = 0$ based on a simple Gauss's law argument, but you might also reasonably second guess yourself since this problem doesn't have translational symmetry. But as it turns out, E really is zero out there, or close to it. Use that with confidence if you need it (I needed it).