Lecture 12

Note Title 2/8/2006

$\nabla^2 V = 0$ with bndry conditions

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$xy + x^2$ doesn't obey this

Assumption $V(x,y,z) = X(x)\,Y(y)\,Z(z)$

plug $V(x,y,z)$ into Laplaces eqn

$$\underbrace{\frac{1}{X(x)}\frac{d^2 X}{dx^2}}_{g(x)} + \underbrace{\frac{1}{Y(y)}\frac{d^2 Y(y)}{dy^2}}_{h(y)} + \underbrace{\frac{1}{Z}\frac{d^2 Z}{dz^2}}_{k(z)} = 0$$

$$g(x) + h(y) + k(z) = 0$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$C_1 + C_2 + C_3 = 0$$

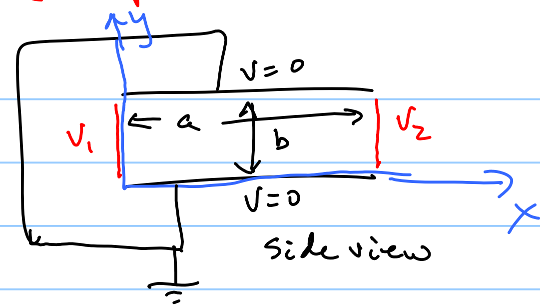
Started with P.D.E. & now have 3 O.D.E.'s

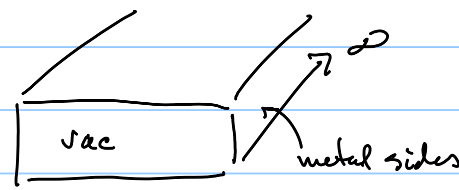
$\frac{1}{V}$

Ex: 2 grnded ||-plate electrodes

other plates are at potential V_1 & V_2 (constant)

$$\text{BC}\begin{cases} x=0 & V=V_1 \\ x=a & V=V_2 \\ y=0,b; & V=0 \end{cases}$$





Field & potential in z direction: do not depend on z

$$Z_1(z) = \text{constant} \Rightarrow C_3 = 0$$

$$\frac{d^2Y}{dy^2} = C_2 Y$$

eigenvalue eqn.

$C_2 < 0$ $\quad Y = A\sin(ky) + B\cos(ky)$ $\quad C_2 = -k^2$

$C_2 > 0$ $\quad Y = A' e^{\sqrt{C_2}\,y} + B' e^{-\sqrt{C_2}\,y}$

$C_2 = 0$ $\quad Y = A'' + B'' y$

Bndy at $y=0$ & $y=b$ is $V=0$ so $\Rightarrow$ $C_2 < 0$ with $k^2 > 0$

$$Y = A\sin ky + B\cos ky \qquad \begin{cases} Y(y=0) = 0 = 0 + B\cos 0 = B \\ Y(y=b) = A\sin kb = 0 \end{cases}$$

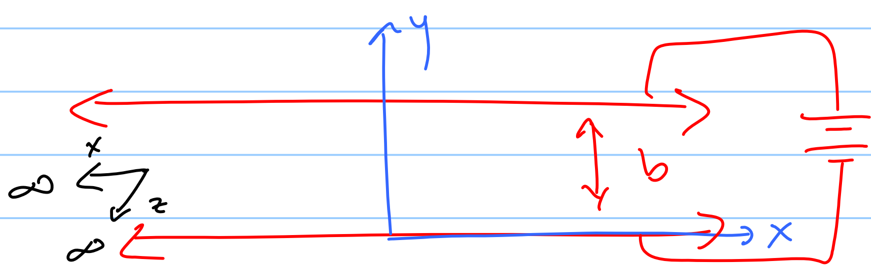
$$kb = n\pi \qquad \underline{n = 1, 2, 3, \ldots}$$

$$\boxed{k = \frac{n\pi}{b}}$$

$$\boxed{Y = A\sin\left(\frac{n\pi}{b}y\right)} \qquad \boxed{Z = \text{constant}}$$

$$C_1 + \underset{-k^2}{\overset{\shortparallel}{C_2}} + \underset{0}{\overset{\shortparallel}{C_3}} = 0 \quad \Rightarrow \quad C_1 = k^2 > 0$$

$$\frac{d^2 X}{dx^2} = k^2 X \qquad \boxed{X(x) = G e^{kx} + H e^{-kx}}$$



$$\frac{1}{Z}\frac{d^2 Z}{dz^2} = C_3, \quad \frac{d^2 Z}{dz^2}: \text{const}, \quad C_3 = 0$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} = C_1 = 0, \quad \frac{d^2 X}{dx^2}: \text{const}$$

$$C_1 + C_2 + C_3 = 0$$

$$\underset{0}{\shortparallel} + \text{?} + \underset{0}{\shortparallel} = 0 \qquad C_2 = 0 \Rightarrow \underline{\text{Linear soln}}$$