HW 9

Note Title 11/19/2006

Snieder 15.5

13.28

$$F(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x) e^{-ikx}\, dx$$

a)

Let $F(k) = C\,\tilde{F}(k)$

$$\Rightarrow f(x) = \int_{-\infty}^{\infty} C\,\tilde{F}(k)\, e^{ikx}\, dk$$

and $C\,\tilde{F}(k) = \frac{1}{2\pi}\int f(x)\, e^{-ikx}\, dx$

So
$$\boxed{\begin{aligned} f(x) &= C\int \tilde{F}\, e^{ikx}\, dk \\ \tilde{F}(k) &= \frac{1}{C2\pi}\int f(x) e^{-ikx}\, dx \end{aligned}}$$

b) C is compl. arbitrary

the prod. of the normalization constants is

$$C \cdot \frac{1}{C 2\pi} = \frac{1}{2\pi}$$

c) $$f(x) = \int_{-\infty}^{\infty} F(k)\, e^{ikx}\, dk$$

$k \Rightarrow -k \qquad dk \to -dk$

$$f(x) = -\int_{+\infty}^{-\infty} \underbrace{F(-k)}_{\equiv \tilde{F}(k)}\, e^{-ikx}\, dk$$

$$= \int_{-\infty}^{\infty} \tilde{F}(k)\, e^{-ikx}\, dk$$

and since

$$\tilde{F}(-k) = F(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx$$

we have $\hat{F}(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\, e^{ikx}\, dx$

d) plane wave

$F\, e^{i(kx-\omega t)}$

as t increases, x must increase in order to keep phase constant

$\phi \equiv kx - \omega t$

$\Delta\phi = k\Delta x - \omega\Delta t$

in order that $\Delta\phi = 0$

we must have $k\Delta x = \omega\Delta t$

$$\boxed{\frac{\Delta x}{\Delta t} = \frac{\omega}{k} = c}$$

e $$f(x,t) = \iint_{-\infty}^{\infty} F(k,\omega)\, e^{i(kx-\omega t)}\, dk\, d\omega$$

$$F(k,\omega) = \left(\frac{1}{2\pi}\right)^2 \iint_{-\infty}^{\infty} f(x,t)\, e^{-i(kx-\omega t)}\, dx\, dt$$

for fixed x $f(x,t)$ is a function of t. so the two fourier trans. can be done independently.

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2)

Details are given in class notes and mathematica NB for week of 10/30/06

2 spec. interferometry

1) $$g(t, m, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t-m)^2/2\sigma^2}$$

$$G(\omega, m, \sigma) = (\;)\, \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} e^{-(t-m)^2/2\sigma^2}$$

either $\frac{1}{2\pi}$ or

$\frac{1}{\sqrt{2\pi}}$

exponent

$$e^{-\underbrace{\left[i\omega t + (t-m)^2/2\sigma^2\right]}}$$

$$\frac{1}{2\sigma^2}\left[(t-m)^2 + i2\sigma^2\omega t\right]$$

$$\frac{1}{2\sigma^2}\left[t^2 - 2mt + m^2 + i2\sigma^2\omega t\right]$$

$$\frac{1}{2\sigma^2}\left[t^2 - t(2m - i2\sigma^2\omega) + m^2\right]$$

$$\frac{1}{2\sigma^2}\left[\left(t-(m-i\sigma^2\omega)\right)^2-(m-i\sigma^2\omega)^2 + m^2\right]$$

$$= \frac{1}{2\sigma^2}\left[\left(t-(\quad)^2\right)+2im\sigma^2\omega+\sigma^4\omega^2\right]$$

$$\Rightarrow e^{-[\qquad\qquad]}$$

$$= e^{-\frac{1}{2\sigma^2}[t-(\ )^2]}\ e^{im\omega}\ e^{\sigma^2\omega^2/2}$$

So $\displaystyle G(\omega) = e^{-im\omega}\ e^{-\sigma^2\omega^2/2}\ \times$

$$\frac{1}{2\pi}\ \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}(\underbrace{t-(\ )}^2)}\,dt$$

this shift has no effect since integ. from $-\infty$ to $+\infty$

$$\int_{-\infty}^{\infty} e^{-z^2/2\sigma^2} \, dz$$

one Last change of variable

$$\frac{z^2}{2\sigma^2} = x^2$$

$$dz = \sqrt{2}\,\sigma \, dx$$

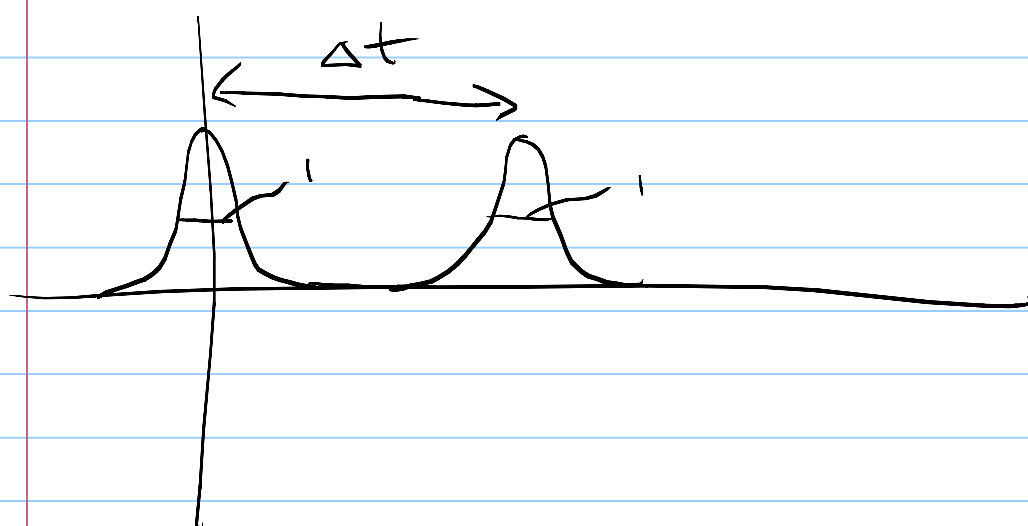
$$\frac{1}{\sqrt{2\sigma^2}} \underbrace{\int_{-\infty}^{\infty} e^{-x^2} \, dx}_{\sqrt{\pi}}$$

Putting this all together we get

$$G(\omega, m, \sigma) = \frac{e^{-i\omega m} \, e^{-\sigma^2 \omega^2 / 2}}{\sqrt{2\pi}}$$

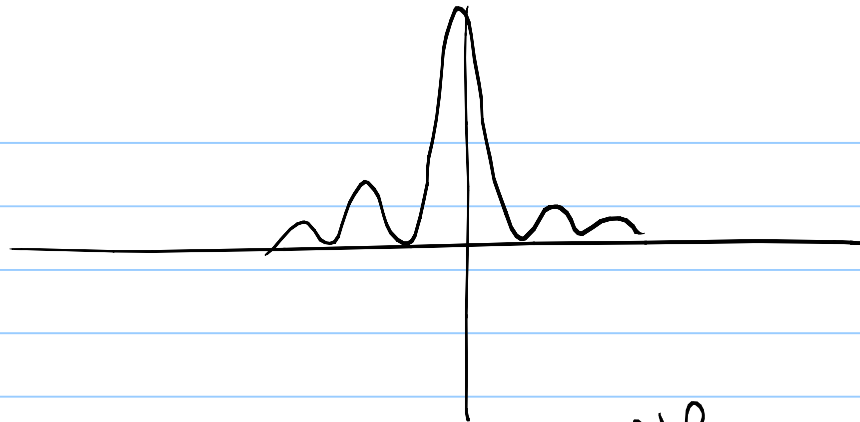
$\hookrightarrow$ scales norm.

2)



mathematic is fine for this

Re



NB phase shift

Im

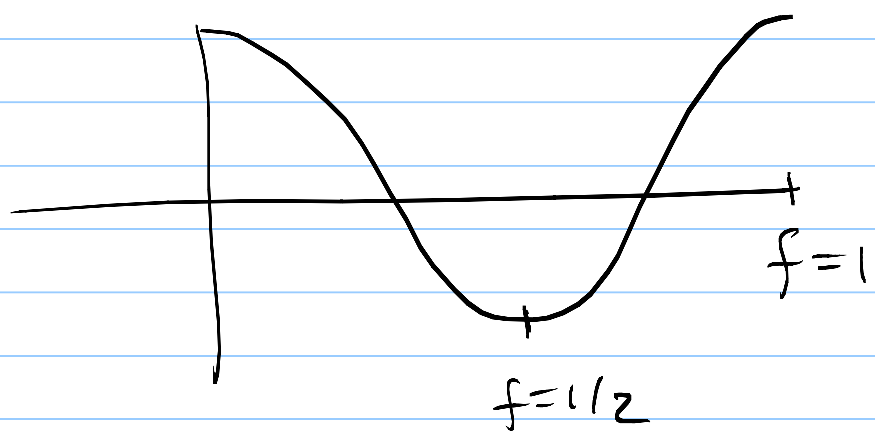
Abs

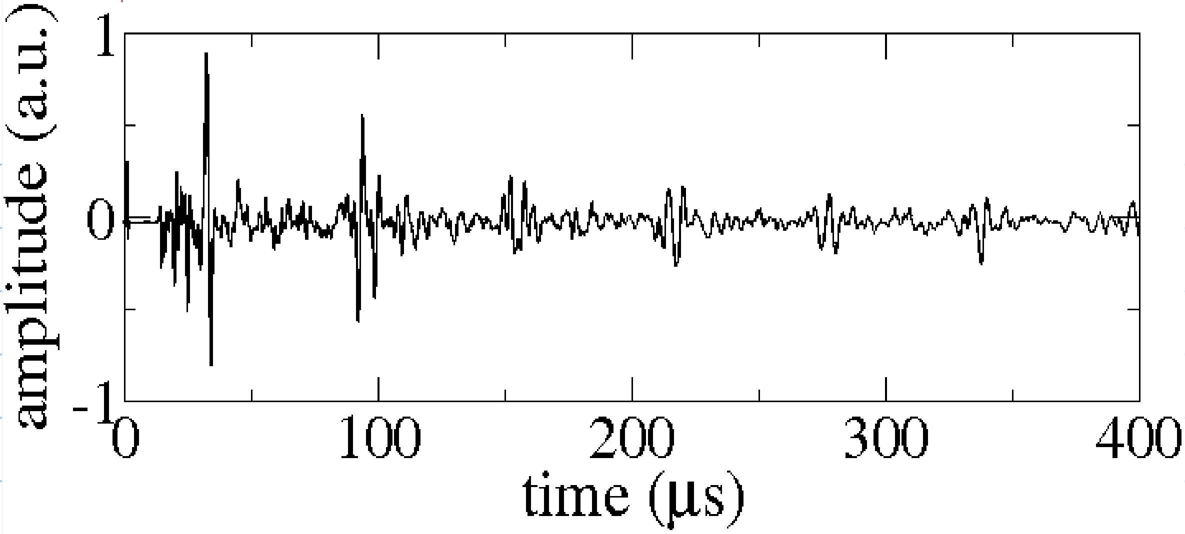
oscillations are governed by $e^{i\omega m}$

$= e^{i 2\pi f m}$

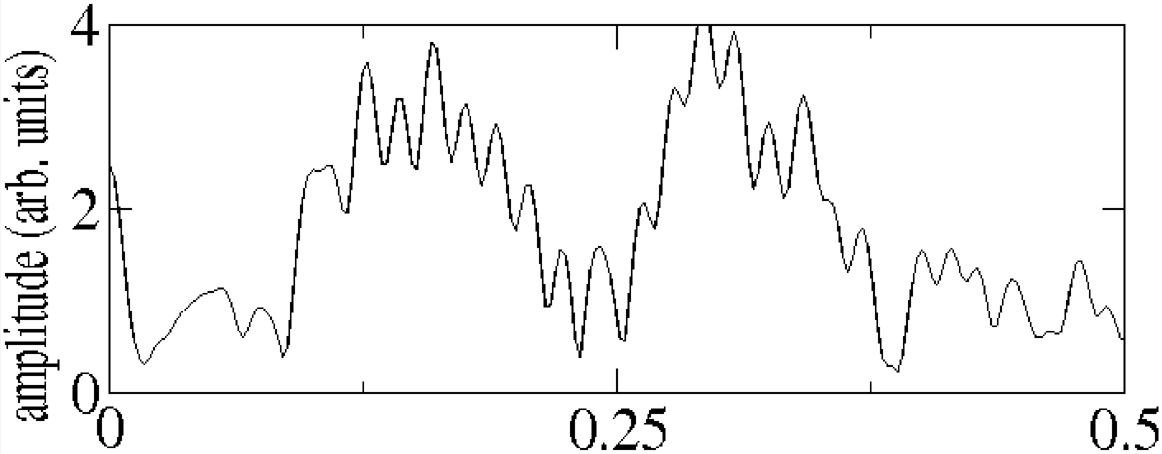
so spacing between peaks is $\frac{1}{m}$

E.g. $m=1$

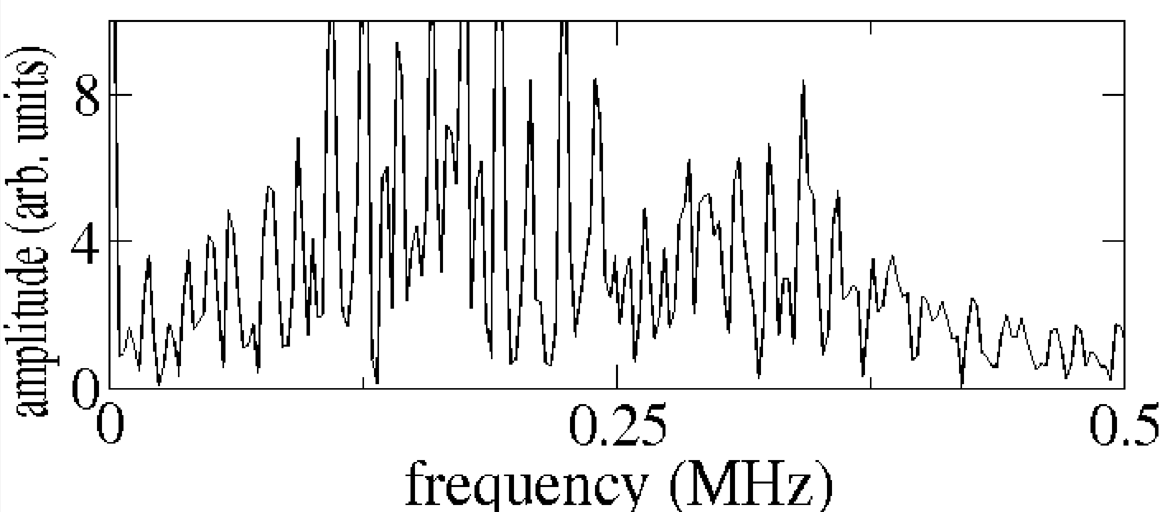




time
series



2 pulses
zero
padded



all
6
pulses.

notice how much sharper the interferences are with more pulses.