Reading: Today: 12.1

Tomorrow: 12.2

What makes a vector a vector?

It has magnitude / direction.
or 3-components.

But you need more.

1) How does it rotate?

Rotations can be written as a similarity transformation

$$\vec{x}' = A\vec{x}$$

rotation matrix

$|A| = 1$ for proper rotations

true rotation

$|A| = -1$ for improper rotations

rotation + odd # of reflections. (parity transformation)

For a true vector, improper rotations flip its sign, but for some "vectors", that's not true.

Say ang. momentum : $\vec{L} = \vec{r} \times \vec{p}$

Something else is time reversal.

$\vec{x}$ even under time reversal (doesn't change for a given value of t.)

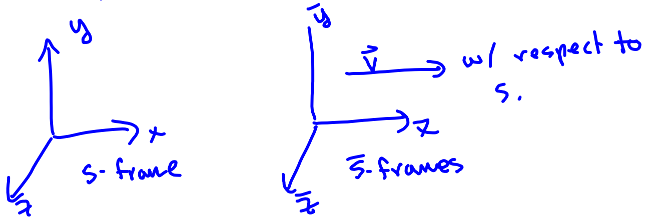
velocity → $\vec{v}$; odd under time reversal (flips sign if you go backward in t).

$\vec{v} = \frac{d\vec{x}}{dt}$; $t \to -t$ flips sign.

Let's make a List

Thing	Rank {How it rotates}	Space inv. parity	t-reversal	ref. frame.
$\vec{x}$ ($\vec{r}$)	1	odd	even	
$\vec{v}$	1	odd	odd	
$\vec{p}$	1	odd	odd	
$\vec{L}$	1	even	odd	
$\vec{F}$	1	odd	even	
$\vec{r}\times\vec{F}$	1	even	even	
Kin. Ene	0	even	even	
Pot. E	0	even	even	
ρ	0	even	even	
$\vec{J}$	1	odd	odd	
$\vec{E}$, $\vec{P}$, $\vec{D}$	1	odd	even	
$\vec{B}$, $\vec{M}$, $\vec{H}$	1	even	odd	
$\vec{S}$	1	odd	odd	
$\overleftrightarrow{T}$	2	even	even	
$\vec{\nabla}$	1	odd	even	

The question is, how do these things transform when changing ref. frames.



The speed of light in any reference frame is the same constant, c.
{i.e. for all observers}.

Consequence is

$$\bar{x} = \gamma(x - vt) + x_0 \qquad x = \gamma(\bar{x} + v\bar{t}) - x_0$$

$$\bar{y} = y + y_0 \qquad y = \bar{y} - y_0$$

$$\bar{z} = z + z_0 \qquad z = \bar{z} - z_0$$

$$\bar{t} = \gamma\left(t - \frac{v}{c^2}x\right) + t_0 \qquad t = \gamma\left(\bar{t} + \frac{v}{c^2}\bar{x}\right) - t_0$$

v is really v_x, and it has a sign!

For simplicity, at $t=0$, set $\bar{t}=0$ $\quad x=\bar{x}$, $y=\bar{y}$, $z=\bar{z}$

$\rightarrow x_0, y_0, z_0, t_0 = \emptyset$

Title: Aug 4 - 6:15 PM (5 of 10)

What are the consequences?

Simultenaity: two things.

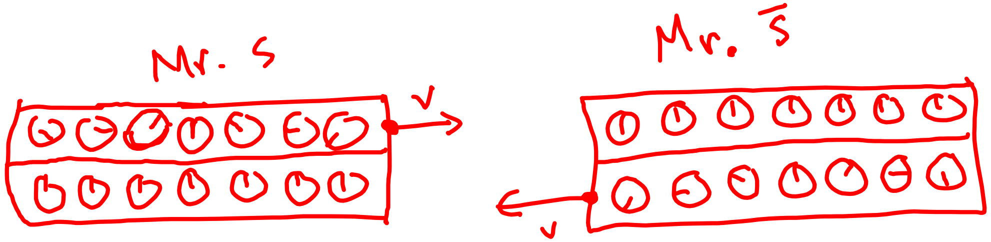
Let's say in frame S that something happens at $t=\phi$ at $x=\pm a$

Question: what does Mr. $\bar{S}$ say?

He says $\bar{t}_1 = \frac{\gamma v}{c^2} a \quad \{x=-a\}$

$$\bar{t}_2 = -\frac{\gamma v}{c^2} a \quad \{x = a\}$$

Both Mr. S, and Mr. $\bar{S}$ set up sinchronized clocks in their frames. What does that look like?



As an exersize for the reader show for a single clock in $\bar{S}$, $\bar{x} = \text{const}$

$\Delta t = \gamma \Delta \bar{t} \Rightarrow \Delta \bar{t} = \frac{1}{\gamma} \Delta t$ ← time dilation

Take $\Delta \bar{x}$ but w/ $\Delta t = \phi$

$\Rightarrow \Delta \bar{x} = \gamma \Delta x$ ← length contraction

4 vectors

$x^\mu = (ct, x, y, z)$ ← contravarient vector

$x_\mu = (-ct, x, y, z)$ ← covariant vector.

Contravarient vectors transform like {when going from $S \to \bar{S}$}

$\beta = v/c$

$\bar{x}^\mu = \Lambda^\mu{}_\nu x^\nu$ $\quad$ $\Lambda^\mu{}_\nu = \begin{Bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$

any contravariant vector not just x

For the reader,

show $\Lambda^\mu{}_\nu = \dfrac{\partial \bar{x}^\mu}{\partial x^\nu}$ using the Lorentz equs from before.

A contravarient vector is one which transforms like $\bar{v}^\mu = \dfrac{\partial \bar{x}^\mu}{\partial x^\nu} v^\nu$

A covarient vector is one which transforms like

$$\bar{v}^\mu = \frac{\partial x^\mu}{\partial \bar{x}^\nu} v^\nu$$

How do I change

$$x^{\mu} \rightarrow x_{\mu}$$

or vice versa

Enter the metric $g_{\mu\nu} = g^{\mu\nu}$

For special relativity (flat space)

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$g_{\mu\nu} a^{\mu} = a_{\nu}$$

Dot products

$a_\mu b^\mu = \underline{\underline{\underline{\text{scalar}}}}$

same in all reference frames.

$$g_{\nu\mu} a^\nu b^\mu$$ {Generalization}

How to transform tensors.

$$\bar{T}^{\mu\nu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\gamma}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\alpha}} T^{\gamma\alpha}$$