9/8/06

Note Title 9/8/2006

$\underbrace{\text{http://mesoscopic.mines.edu/~jscales}}_{\text{my home page (via physics)}}$/Snieder_2.pdf

complex numbers $z = z_r + i z_i$

e^{izt} $= |z| e^{i\varphi}$ ← phase

↑ Amplitude

This works if and only if all operations are linear.

E.g. Solve Maxwel for $\vec{E}$

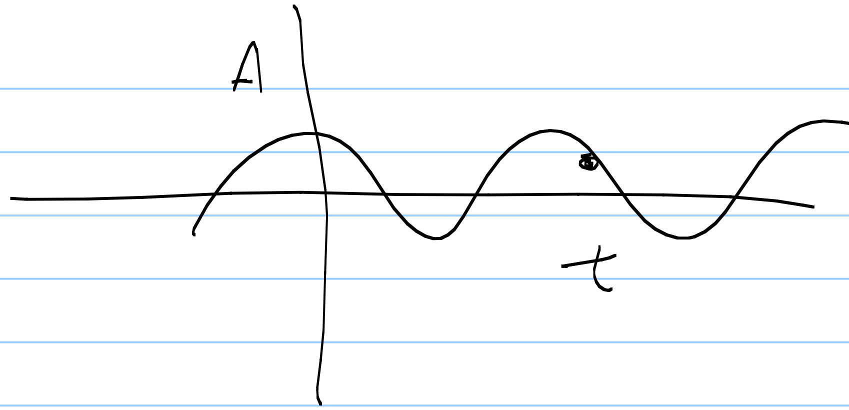
assume $\vec{E} = \vec{E}_0 e^{i\omega t}$

. . . . solve

Energy density $\equiv$ Intensity $= |\vec{E}|^2 = \vec{E}\vec{E}^*$

Cannot simply take the real part

nonlinear operation



$Ae^{i\omega t}$ —

ωt

$E = E_o e^{i\omega t}$ Assume

solve for E_o, ω

When can we assume. Solutions are complex?

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Examples of operators

$$\frac{d}{dx} : f \to f'$$

$$\int : f \to \int f(x)\,dx$$

generic notation $L(f) = g$

if and only if

L is linear $\iff L(f_1 + f_2) = L(f_1) + L(f_2)$

and $L(cf) = cL(f)$ where

c is a constant.

Usually these are combined:

$$L(cf_1 + f_2) = cL(f_1) + L(f_2)$$

for linear operators

Complicated example:

$$L(x(t)) = \ddot{x} + \gamma \dot{x} + \omega_0^2 x$$

is this a Linear operator?

use Def suppose $x_1(t)$, $x_2(t)$ two diff. functions

$$L(cx_1 + x_2) = (c\ddot{x}_1 + \ddot{x}_2) + \gamma(c\dot{x}_1 + \dot{x}_2) + \omega_0^2(cx_1 + x_2)$$

$$= \underbrace{c[\ddot{x}_1 + \gamma \dot{x}_1 + \omega_0^2 x_1]}_{c\,L(f_1)} + \underbrace{[\ddot{x}_2 + \gamma \dot{x}_2 + x_2]}_{L(f_2)}$$

$$c\,L(f_1) \quad + \quad L(f_2)$$

✓

Damped, forced Simple Harmonic Osc.

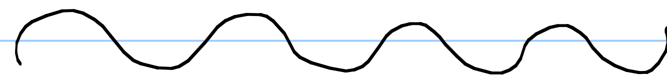
$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = F/m = F(t)/m \overset{\text{simplify}}{=} \frac{F_0}{m}\cos\omega t$$

Assume x, F are complex

$$= \frac{F_0}{m} e^{i\omega t}$$

Input ω $\longrightarrow$ $\boxed{\text{Linear System}}$ $\longrightarrow$ Output Same ω

what's diff is Amplitude / Phase

Input  ω

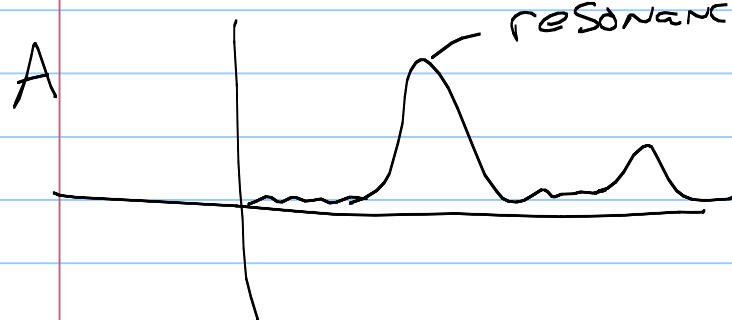
output  ω

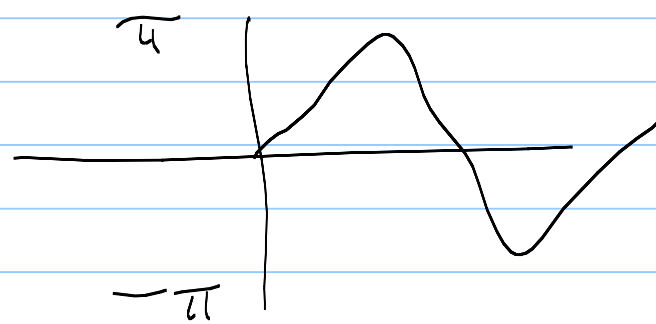
$\}$ same freq

diff. amplitudes + phase

This [illegible]

Sweep over f, measure $A(f)$, $\varphi(f)$





$\omega_0 = \sqrt{\frac{k}{m}}$

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} e^{i\omega t}$$

Guess $x(t) = x_0 e^{i\omega t}$

Plug in

$$-\omega^2 x_0 e^{i\omega t} + i\gamma\omega x_0 e^{i\omega t} + \omega_0^2 x_0 e^{i\omega t} = F_0/m \, e^{i\omega t}$$

cancel the $e^{i\omega t}$, factor out x_0

$$\Rightarrow \quad (-\omega^2 + \omega_0^2 + i\gamma\omega) x_0 = F_0/m$$

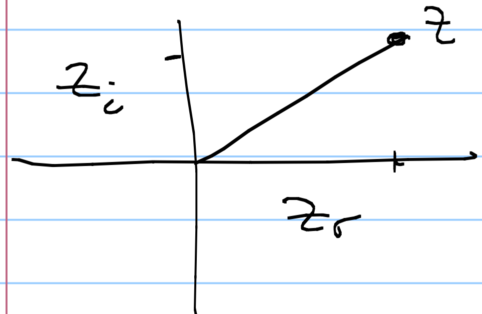
$$\boxed{x_0 = \frac{F_0/m}{\omega_0^2 - \omega^2 + i\gamma\omega}}$$

We want this to be $Ae^{i\varphi}$

$$X(\omega) = \frac{F_0/m}{((\omega_0^2-\omega^2) + i\gamma\omega)}$$

$$|(\omega_0^2-\omega^2) + i\gamma\omega| = \sqrt{(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}$$

$$\mathrm{Arg}((\omega_0^2-\omega^2) + i\gamma\omega) = \mathrm{Arctan}\left[\frac{\gamma\omega}{(\omega_0^2-\omega^2)}\right]$$



$z = z_r + iz_i$

$$|z| = \sqrt{z_r^2 + z_i^2}$$

$$\mathrm{Arg}(z) = \tan^{-1}(z_i/z_r)$$

$$|(\omega_0^2-\omega^2)+i\gamma\omega| = \sqrt{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}$$

$$\operatorname{Arg}\left[(\omega_0^2-\omega^2)+i\gamma\omega\right] = \arctan\left[\frac{\gamma\omega}{(\omega_0^2-\omega^2)}\right]$$

$$X(\omega) = \underbrace{\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}}}_{\text{Lorentzian}} e^{-\tan^{-1}\left(\frac{\gamma\omega}{(\omega_0^2-\omega^2)}\right)} e^{i\omega t}$$