

Propagation of non monochromatic light.

single frequency:

$$E(t) = E_0 e^{-i\omega_0 t}$$

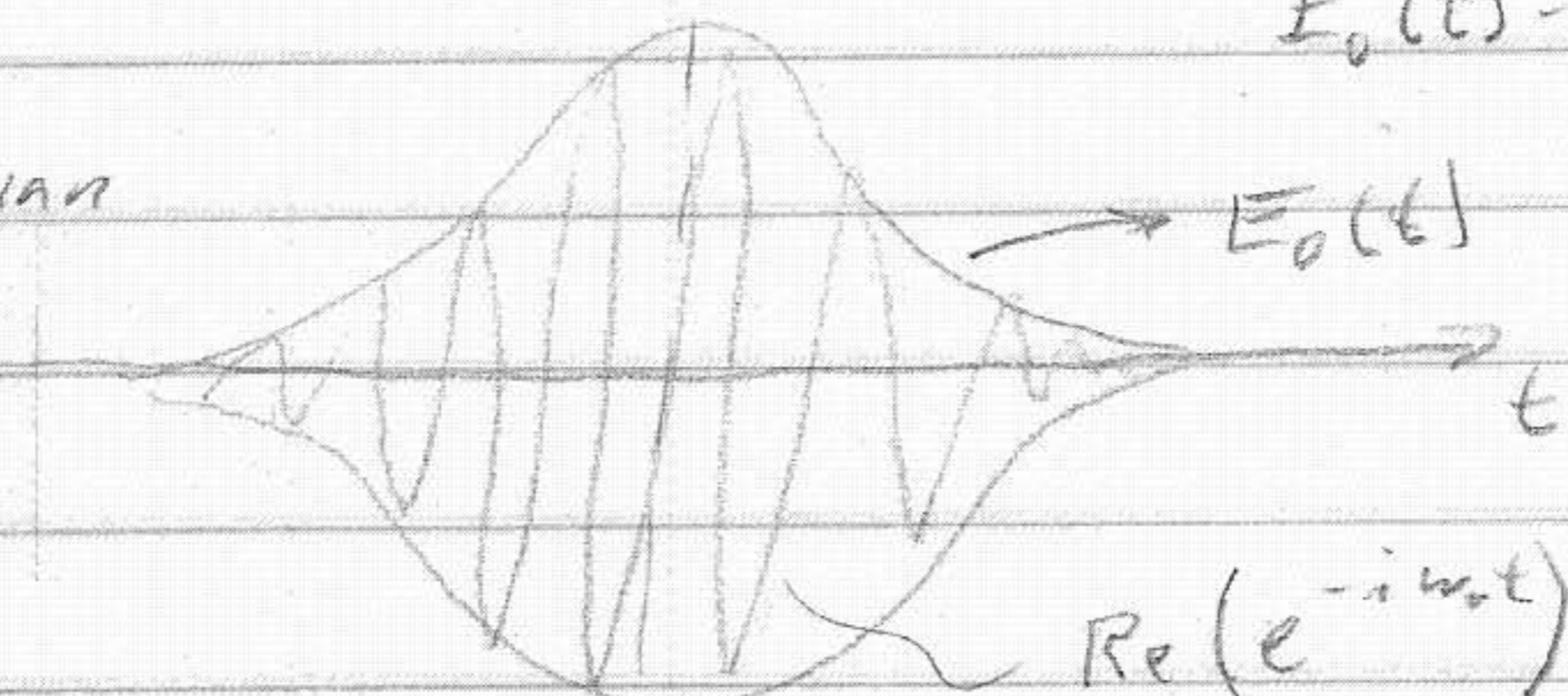
quasi-monochromatic:

$$E(t) = E_0(t) e^{-i\omega_0 t}$$

$\omega_0 = \text{carrier freq.}$

$E_0(t) = \text{envelope.}$

ex. Gaussian



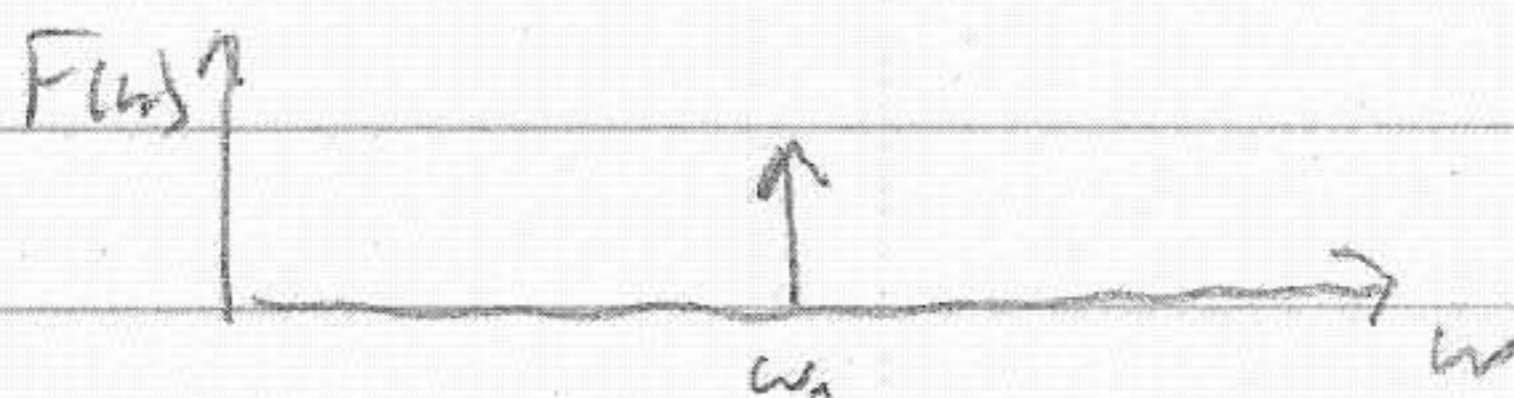
Any modulation in the time domain requires a mixture of frequencies. Use Fourier transforms to get spectrum:

Define $\mathcal{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$

$$\mathcal{F}^{-1}\{F(\omega)\} = f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$$

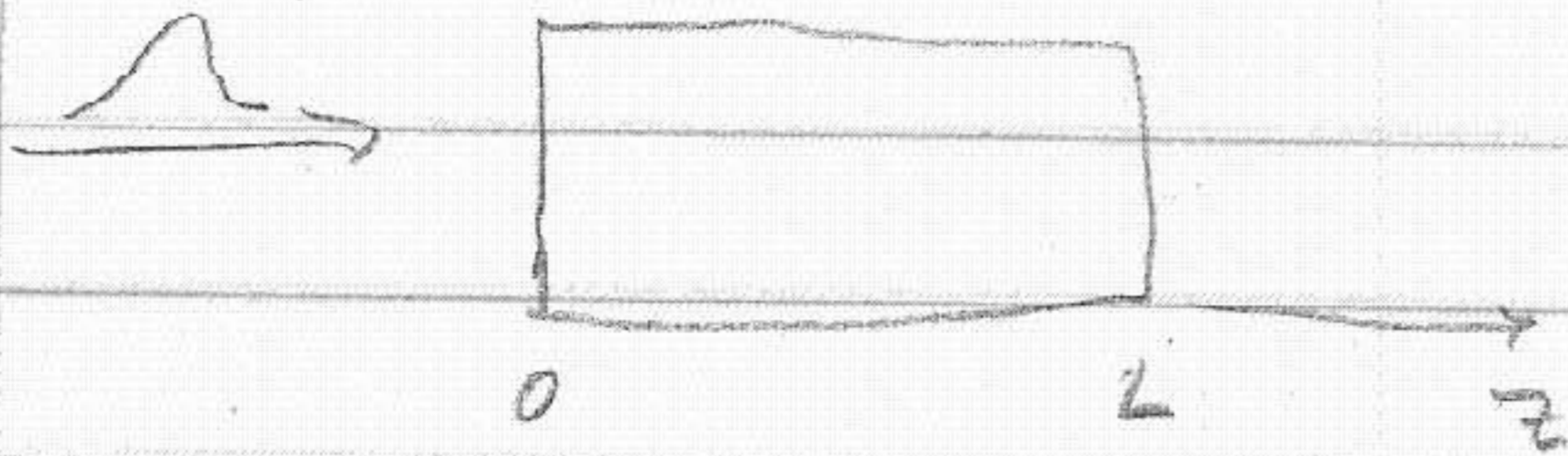
- note opposite signs in $\mathcal{F}\{\}$ and $\mathcal{F}^{-1}\{\}$
- choice in signs is consistent with $e^{-i\omega_0 t}$ convention

$$\mathcal{F}\{e^{-i\omega_0 t}\} = \int_{-\infty}^{\infty} e^{i(\omega - \omega_0)t} dt = 2\pi \delta(\omega - \omega_0)$$



- note placement of 2π factor.

Dispersion



By travelling from $z=0$ to $z=L$,

$$E(z=0) e^{ik_0 L}$$

since $n = n(\omega)$ we must apply this in the freq. domain

$$\tilde{E}_{out}(\omega) = \tilde{E}_{in}(\omega) e^{i \frac{\omega}{c} n(\omega) L}$$

note that if $n(\omega)$ is real,

$|E_{out}(\omega)|^2$ is unchanged by medium
i.e. no absorption

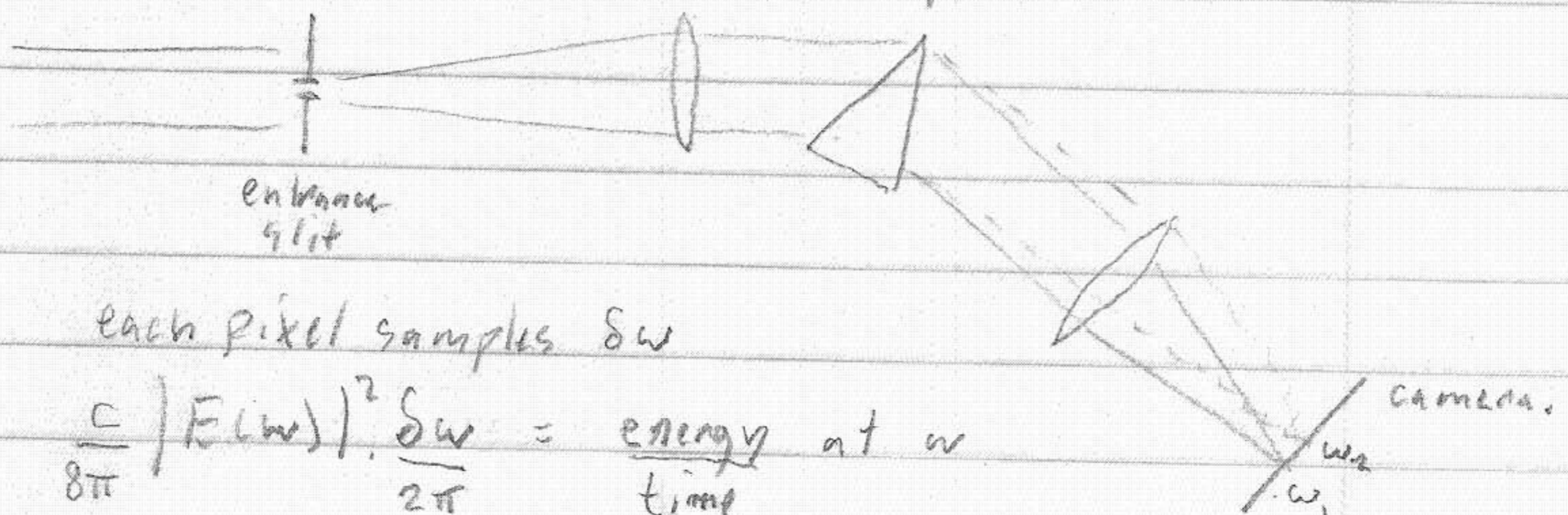
$\frac{\omega}{c} n(\omega) L$ is just a phase shift, depends on ω .

$\equiv \phi(\omega)$ spectral phase.

In a linear system, each frequency component propagates on its own.

$|E_{out}(\omega)|^2 \propto$ spectral intensity

this is what is measured by a spectrometer:



each pixel samples $\Delta\omega$

$$\frac{c}{8\pi} |E(\omega)|^2 \frac{\Delta\omega}{2\pi} = \frac{\text{energy}}{\text{time}} \text{ at } \omega$$

Gaussian pulse propagating through a dispersive medium

$$\text{let } E_i(t) = E_0 a(t) e^{-i\omega_0 t}$$

$$\text{with } a(t) = \exp(-t^2/\tau^2)$$

here no extra phases

$$\text{calc. spectrum: } A(\omega) = \mathcal{F}\{a(t) e^{-i\omega_0 t}\}$$

$$1) \mathcal{F}\{e^{-t^2/\tau^2}\} = \int_{-\infty}^{\infty} e^{-t^2/\tau^2 + i\omega t} dt$$

complete square in exponent to get to form:

$$\int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi}$$

$$-\frac{1}{\tau^2}(t^2 - i\omega\tau^2 t) = -\frac{1}{\tau^2}\left[\left(t - \frac{i\omega\tau^2}{2}\right)^2 + \frac{\omega^2\tau^4}{4}\right]$$

$$\rightarrow e^{-\frac{\omega^2\tau^2}{4}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{\tau^2}} dt$$

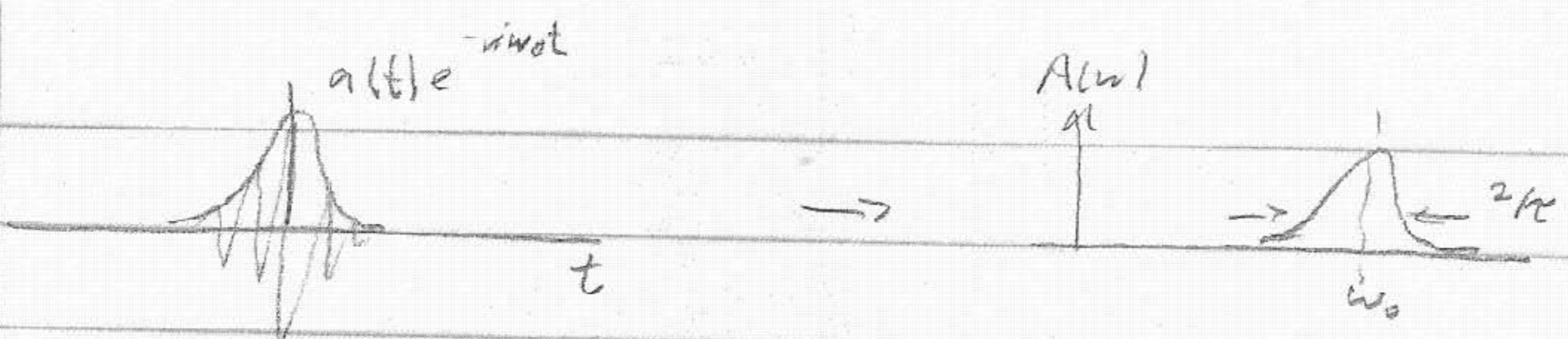
$$\text{let } u = \left(t - \frac{i\omega\tau^2}{2}\right)/\tau \quad du = \frac{1}{\tau} dt$$

$$\mathcal{F}\{e^{-t^2/\tau^2}\} = \tau \cdot \sqrt{\pi} e^{-\omega^2\tau^2/4} \quad \text{another Gaussian.}$$

2) use shift theorem:

$$\int f(t) e^{-i\omega_0 t} e^{i\omega t} dt = F(\omega - \omega_0)$$

$$\therefore A(\omega) = \sqrt{\pi}\tau^2 e^{-\frac{(\omega - \omega_0)^2\tau^2}{4}}$$



in t : $1/e$ half width = τ

in ω : " = $2/\tau$

$\Delta t \Delta \omega = 2$ uncertainty principle

QM: photon energy = $\hbar \omega$

$\Delta E \Delta t = 2\hbar$ note QM definition of width is different.

photon counting: $\hbar \omega$ can be anywhere w/in range $\Delta \omega$
 individual photons share same wavefunction as packet

$$E \sim \psi$$

$|E|^2 \sim |\psi|^2$ energy density \sim probability density

$E(t)$ and $E(\omega)$ are different representations of same signal

\therefore expect the same total energy:

$$\int |E(t)|^2 dt = \frac{1}{2\pi} \int |E(\omega)|^2 d\omega$$

\hookrightarrow comes in from proof

this is Parseval's thm.

check:

$$\int E_0^2 e^{-2t^2/\tau^2} dt = E_0^2 \frac{\tau}{\sqrt{2}} \int e^{-u^2} du = E_0^2 \tau \sqrt{\frac{\pi}{2}}$$

$$\frac{1}{2\pi} \int E_0^2 \pi \tau^2 e^{-(\omega-\omega_0)^2 \tau^2/2} d\omega = \frac{\tau^2}{2} E_0^2 \sqrt{\frac{2}{\tau^2}} \sqrt{\pi} \quad \checkmark$$

$\frac{c}{4\pi} |E(t)|^2 =$ intensity (time dependent) = $I(t)$ e.g. W/cm^2

$\int I(t) dt =$ energy fluence e.g. J/cm^2