MATH348 - July 27, 2009 **NAME**:

Exam II - 50 Points

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) True/False and Short Response

   (a) Mark each statement as either true or false. No justification is needed.

   i. Suppose f is a periodic function such that $f(-x) = -f(x)$. The Fourier series representation of f will have only cosine functions.

   ii. Every real Fourier series can be written as a complex Fourier series.

   iii. The periodic extension, $f^*(x)$, of $f(x)$ is unique.

   iv. The Fourier transform of a function with even symmetry has an odd symmetry.

   v. If a periodic function is neither odd nor even then the Fourier series representation will have both sine and cosine functions.

   (b) Explain why every function defined on a bounded domain of $\mathbb{R}$ has a Fourier series representation.

   (c) How is the Fourier integral related to the Fourier series? What is the purpose of each?

2. (10 Points) Given the following integrals,

$$5 = \int_{-\pi}^{\pi} f(x)dx, \qquad \frac{4}{n\pi}(1-\cos(2n\pi)) = \int_{-\pi}^{\pi} f(x)\cos(nx)dx, \quad \frac{2}{n^2}\sin(n\pi) = \int_{-\pi}^{\pi} f(x)\sin(nx)dx,$$

$$i\frac{(-1)^n}{n} = \int_{-\pi}^{\pi} g(x)e^{-inx}dx, \qquad \frac{e^{in\pi}-e^{-in\pi}}{4\pi} = \int_{-\pi}^{\pi} g(x)dx, \qquad \frac{e^{i\omega}-e^{-i\omega}}{2i\omega} = \int_{-\infty}^{\infty} h(x)e^{-i\omega x}dx.$$

(a) Calculate the real Fourier series of $f(x)$.

(b) Calculate the complex Fourier series of $g(x)$.

(c) Calculate the Fourier transform of $h(x)$.

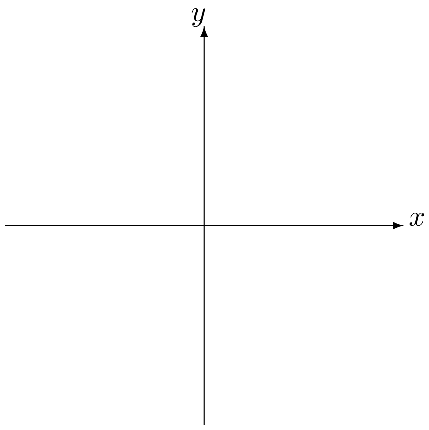
(d) Determine the symmetry of the function $f(x)$.

(e) Determine the symmetry of the function $g(x)$.

(f) Determine the symmetry of the function $h(x)$.

(g) Calculate the real Fourier series representation of $g(x)$.

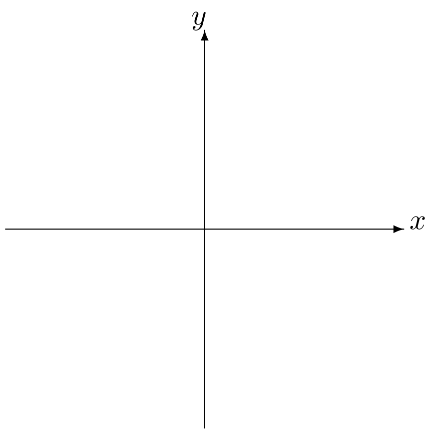
3. (10 Points) Below is the graph of the function f.



(a) Find the Fourier cosine half-range expansion of f.

(b) Find the Fourier sine half-range expansion of f.

(c) Graph the Fourier cosine half-range expansion of f with a dashed line and the Fourier sine half-range expansion with a solid line on the axes below.



4. (10 Points) Find the complex Fourier series representation of $f(x) = \pi$, $x \in (-\pi, \pi)$, where $f(x+2\pi) = f(x)$.

5. (10 Points) Suppose that f is given as,

$$f(x) = \begin{cases} x+1, & -1 \leq x < 0 \\ 1-x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} . \tag{1}$$

Calculate the complex Fourier transform of f. Noting the identity, $2\sin^2(y) = 1 - \cos(2y)$, simplify as much as possible.

6. (Extra Credit)

(a) Let $\hat{f}(\omega) = \sum_{n=-\infty}^{\infty} \sqrt{2\pi} c_n \delta(\omega - n)$. Find the inverse Fourier transform of $\hat{f}$.

(b) Suppose that $c_n = \dfrac{e^{in\pi} - e^{-in\pi}}{n}$ for $n \neq 0$ and $c_0 = \pi$. Graph the function $f(x) = \mathfrak{F}^{-1}\left\{\hat{f}\right\}$ below.