MACS 332 - July 26, 2006 **NAME:**
Quiz II - 30 minutes

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. Please enclose your final answers in boxes.

1. (10 Points) Define $M_{2\times 2}$ as the vector space of all two-by-two matrices with real entries. Let H be the sub set of all real matrices of the form $\mathbf{A} = \begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$. Is H a subspace of $M_{2\times 2}$? Justify your response.

Let $A, B \in H$, $c \in \mathbb{R}$, $A = \begin{bmatrix} a_1 & 0 \\ c_1 & d_1 \end{bmatrix}$, $B = \begin{bmatrix} a_2 & 0 \\ c_2 & d_2 \end{bmatrix}$, $a_1, a_2, c_1, c_2, d_1, d_2 \in \mathbb{R}$

Then

$A + cB = \begin{bmatrix} a_1 & 0 \\ c_1 & d_1 \end{bmatrix} + c\begin{bmatrix} a_2 & 0 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 + ca_2 & 0 \\ c_1 + cc_2 & d_1 + cd_2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$, $a, c, d \in \mathbb{R}$

$\Rightarrow A + cB \in M_{2\times 2}$

Note, Let $a, b, c \in \mathbb{R}$ s.t. $a = b = c = 0$.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in H.$ $\Rightarrow$ H is a subspace of $M_{2\times 2}$.

2. (10 Points) Which of the following matrices are diagonalizable?

$$\checkmark\ \mathbf{A} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}, \quad \times\ \mathbf{B} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad \times\ \mathbf{C} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Explain.

Eigenvalues of A. are $2, 2, 3 = \lambda$

Case $\lambda = 2$

$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$

$\Rightarrow x_3 = 0$, $x_1 =$ free, x_2 free $\Rightarrow \vec{x} = x_2\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Case $\lambda = 3$

$\left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\Rightarrow x_1 = x_2 = 0$, x_3 free $\Rightarrow \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_3$

B, $\lambda = 2, 3$

Case $\lambda = 2$

$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$

$\Rightarrow x_1$ free, $x_3 = x_3 = 0$

$\Rightarrow \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_1$

Case $\lambda = 3$

$\left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_3$

3. (10 Points) Given that,

$$\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}.$$

Determine a diagonal decomposition, $\mathbf{PDP}^{-1}$, of $\mathbf{A}$.

$\det(A - \lambda I) = 0 \Rightarrow (2-\lambda)(-\lambda) + 0 = 0 \Rightarrow \lambda = 0,\ \lambda = 2$

Case $\lambda = 0$:

$\left[\begin{array}{cc|c} 2 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = \text{free} \end{array} \Rightarrow \vec{x} = x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Case $\lambda = 2$:

$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & -2 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 2x_2 \\ x_2 \text{ free} \end{array} \Rightarrow \vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} x_2$

$\Rightarrow P = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & \\ & 2 \end{bmatrix} \quad P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$

$\frac{1}{2}\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix} =$

$= \frac{1}{2}\begin{bmatrix} 0 & 4 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix} =$

$= \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} = A$ ✓

4. (10 Points) Let λ be an eigenvalue of the invertible matrix $\mathbf{A}$. Show that λ^{-1} is an eigenvalue of $\mathbf{A}^{-1}$.

If λ is an Eigenvalue of A ~~then~~ And A^{-1} Exists then

$Ax = \lambda x \Leftrightarrow A^{-1}Ax = \lambda A^{-1}x \Leftrightarrow x = \lambda A^{-1}x \Leftrightarrow A^{-1}x = \frac{1}{\lambda}x$

$\Rightarrow \lambda^{-1}$ is an Eigenvalue of A^{-1}!