Problem 7.17

(a) The field inside the solenoid is $B = \mu_0 nI$. So $\Phi = \pi a^2 \mu_0 nI \Rightarrow \mathcal{E} = -\pi a^2 \mu_0 n (dI/dt)$.

In magnitude, then, $\mathcal{E} = \pi a^2 \mu_0 nk$. Now $\mathcal{E} = I_r R$, so $\boxed{I_{\text{resistor}} = \dfrac{\pi a^2 \mu_0 nk}{R}.}$

B is to the right and increasing, so the field of the loop is to the *left*, so the current is counterclockwise, or $\boxed{\text{to the right,}}$ through the resistor.

(b) $\Delta\Phi = 2\pi a^2 \mu_0 nI$; $I = \dfrac{dQ}{dt} = \dfrac{\mathcal{E}}{R} = -\dfrac{1}{R}\dfrac{d\Phi}{dt} \Rightarrow \Delta Q = \dfrac{1}{R}\Delta\Phi$, in magnitude. So $\boxed{\Delta Q = \dfrac{2\pi a^2 \mu_0 nI}{R}.}$

Problem 7.18

$$\Phi = \int \mathbf{B}\cdot d\mathbf{a};\ \mathbf{B} = \frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\phi}};\ \Phi = \frac{\mu_0 I a}{2\pi}\int_a^{2a}\frac{ds}{s} = \frac{\mu_0 I a \ln 2}{2\pi};\ \mathcal{E} = I_{\text{loop}}R = \frac{dQ}{dt}R = -\frac{d\Phi}{dt} = -\frac{\mu_0 a \ln 2}{2\pi}\frac{dI}{dt}.$$

$$dQ = -\frac{\mu_0 a \ln 2}{2\pi R}\,dI \Rightarrow \boxed{Q = \frac{I\mu_0 a \ln 2}{2\pi R}.}$$

The field of the wire, at the square loop, is *out of the page*, and *decreasing*, so the field of the induced current must point out of page, within the loop, and hence the induced current flows $\boxed{\text{counterclockwise.}}$

Problem 7.19

In the quasistatic approximation, $\mathbf{B} = \begin{cases} \frac{\mu_0 NI}{2\pi s}\hat{\boldsymbol{\phi}}, & \text{(inside toroid);} \\ 0, & \text{(outside toroid)} \end{cases}$

(Eq. 5.58). The flux around the toroid is therefore

$$\Phi = \frac{\mu_0 NI}{2\pi}\int_a^{a+w}\frac{1}{s}h\,ds = \frac{\mu_0 NIh}{2\pi}\ln\left(1+\frac{w}{a}\right) \approx \frac{\mu_0 Nhw}{2\pi a}I.\quad \frac{d\Phi}{dt} = \frac{\mu_0 Nhw}{2\pi a}\frac{dI}{dt} = \frac{\mu_0 Nhwk}{2\pi a}.$$

The electric field is the same as the *magnetic* field of a circular current (Eq. 5.38):

$$\mathbf{B} = \frac{\mu_0 I}{2}\frac{a^2}{(a^2+z^2)^{3/2}}\hat{\mathbf{z}},$$

with (Eq. 7.18)

$$I \to -\frac{1}{\mu_0}\frac{d\Phi}{dt} = -\frac{Nhwk}{2\pi a}.\quad \text{So } \mathbf{E} = \frac{\mu_0}{2}\left(-\frac{Nhwk}{2\pi a}\right)\frac{a^2}{(a^2+z^2)^{3/2}}\hat{\mathbf{z}} = \boxed{-\frac{\mu_0}{4\pi}\frac{Nhwka}{(a^2+z^2)^{3/2}}\hat{\mathbf{z}}.}$$

Problem 7.20

(a) From Eq. 5.38, the field (on the axis) is $\mathbf{B} = \frac{\mu_0 I}{2}\frac{b^2}{(b^2+z^2)^{3/2}}\hat{\mathbf{z}}$, so the flux through the little loop (area πa^2) is $\boxed{\Phi = \dfrac{\mu_0 \pi I a^2 b^2}{2(b^2+z^2)^{3/2}}.}$

(b) The field (Eq. 5.86) is $\mathbf{B} = \frac{\mu_0}{4\pi}\frac{m}{r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}})$, where $m = I\pi a^2$. Integrating over the spherical "cap" (bounded by the big loop and centered at the little loop):

$$\Phi = \int \mathbf{B}\cdot d\mathbf{a} = \frac{\mu_0}{4\pi}\frac{I\pi a^2}{r^3}\int (2\cos\theta)(r^2\sin\theta\,d\theta\,d\phi) = \frac{\mu_0 I a^2}{2r}2\pi\int_0^{\bar\theta}\cos\theta\sin\theta\,d\theta$$

where $r = \sqrt{b^2+z^2}$ and $\sin\bar\theta = b/r$. Evidently $\Phi = \frac{\mu_0 I\pi a^2}{r}\left.\frac{\sin^2\theta}{2}\right|_0^{\bar\theta} = \boxed{\dfrac{\mu_0\pi I a^2 b^2}{2(b^2+z^2)^{3/2}},}$ the same as in (a)!!

(c) Dividing off I ($\Phi_1 = M_{12}I_2$, $\Phi_2 = M_{21}I_1$): $\boxed{M_{12} = M_{21} = \dfrac{\mu_0\pi a^2 b^2}{2(b^2+z^2)^{3/2}}.}$