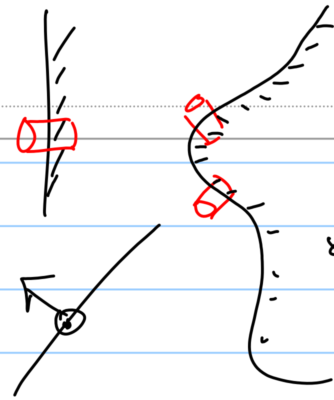
Note Title 7/18/2006



comsol for solving PDE's

γv ↑ ↓ mg

$\overset{v}{\longleftrightarrow}$ qE

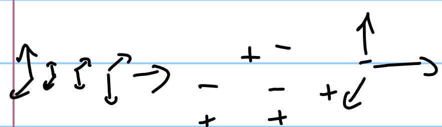
Ohm's law $J \propto v$

$J = \sigma E$

EM fields in conductors

- look at a microscopic model for $\vec{J} = \rho \vec{v}$

ρ: $\frac{\text{charge}}{m^3}$ $\quad v$: $\frac{m}{s}$



Look at charge at $x = 0$

collision with other charges

$$\boxed{\underset{E_0 e^{-i\omega t}}{qE} - m\gamma \frac{d\tilde{y}}{dt} - \cancel{Kr} = m\frac{d^2\tilde{y}}{dt}}$$

spring

steady state

assume $\tilde{y} = \tilde{y}_0 e^{-i\omega t}$

phase

$$q\vec{E} + \underbrace{q\vec{v}\times\vec{B}}_{} = qE + qv\frac{E}{c} = qE\left(1 + \frac{v}{c}\right)$$

$$\frac{E}{c} = B$$

$$qE_0 e^{-i\omega t} - m\overset{1/\tau}{\gamma}\frac{d(\tilde{y}_0 e^{-i\omega t})}{dt} = m\frac{d^2}{dt^2}(\tilde{y}_0 e^{-i\omega t}) \qquad (-i\omega)^2 = -\omega^2$$

$$qE_0 \cancel{e^{-i\omega t}} + i\omega m\gamma\tilde{y}_0 \cancel{e^{-i\omega t}} = -my_0\omega^2\tilde{y}_0 \cancel{e^{-i\omega t}}$$

$$\boxed{\tilde{y}_0 = \frac{e/m}{\underbrace{\omega^2 + i\gamma\omega}}E_0}$$

complex: phase shift between driving term & response

$y = \hat{y}_0 \underbrace{e^{-i\omega t}}_{\text{driving}}$

$$\vec{J} = \rho\vec{v} = \rho\frac{d\hat{y}}{dt}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} + \cancel{\mu_0\frac{\partial^2 \vec{P}}{\partial t^2}} + \mu_0\frac{\partial\vec{J}}{\partial t}$$

particles / vol — molecules

free electrons / molecule

$$J = \underbrace{-eNf}_{\rho}\frac{d\hat{y}}{dt} = -eNf\frac{e/m}{\omega^2 + i\gamma\omega}E_0(-i\omega)e^{-i\omega t} = \frac{-\frac{Ne^2}{m}f(-i\cancel{\omega})}{\underbrace{i\cancel{\omega}(\gamma - i\omega)}}\underbrace{E_0 e^{-i\omega t}}$$

$$\tilde{\vec{J}} \longrightarrow \tilde{\sigma} \qquad \vec{E}$$

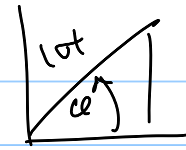
Not Ohms law

(1) $\sigma(\omega)$ conductivity depends on freq.

(2) complex $\Rightarrow$ phase shift between applied field E & the response σ

$$\hat{J} = \tilde{\sigma} E_0 e^{-i\omega t} = |\hat{\sigma}| e^{i\varphi} e^{-i\omega t} = |\tilde{\sigma}| e^{-i(\omega t - \varphi)}$$

$$Re(\hat{J}) = |\hat{\sigma}| \cos(\omega t - \varphi)$$



$$\tan\varphi = \frac{Im\,\hat{\sigma}}{Re\,\tilde{\sigma}} \qquad \gamma = \frac{1}{\tau}$$

$$\underbrace{\frac{-\frac{Ne^2 f}{m}(\cancel{-i\omega})}{\cancel{i\omega}(\gamma - i\omega)}} = \tilde{\sigma} \; ; \quad \sigma_0 = \tilde{\sigma}\Big|_{\omega\to 0} = \frac{+\frac{Ne^2 f}{m}}{\gamma} = \frac{Ne^2 f}{m}\tau$$

D.C. conductivity (Griffiths)

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} + \mu_0\cancel{\frac{\partial^2\vec{P}}{\partial t^2}} + \mu_0\frac{\partial\vec{J}}{\partial t}, \qquad \vec{J} = \tilde{\sigma}\vec{E}$$

particles molecules

$$\underset{\frac{\partial^2 E}{\partial x^2}}{\nabla^2\vec{E}} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \frac{\mu_0\sigma_0\gamma}{\gamma - i\omega}\frac{\partial\vec{E}}{\partial t}$$ PDE (wave eqn)

Dispersion relation obtained by substituting $E = E_0 e^{i(kx-\omega t)}$

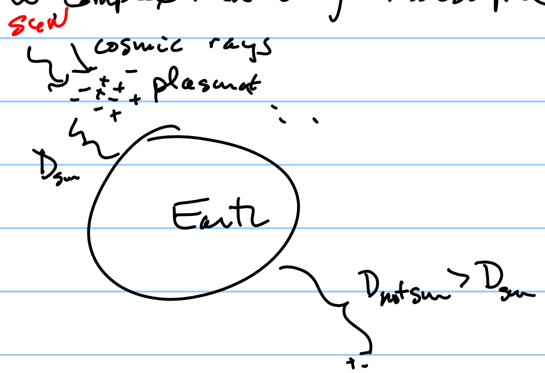
$$\boxed{-k^2 = -\frac{\omega^2}{c^2} + \frac{\mu_0 \sigma_0 \gamma\, i\omega}{\gamma - i\omega}}$$

from relation can get $v_g = \frac{d\omega}{dk}$

phase vel $= \frac{\omega}{k}$

attenuation occurs when k is complex Then imag $\Rightarrow$ absorption

Examples: ionosphere



typically σ is pure imaginary

$$\sigma = \frac{\sigma_0 \gamma}{\gamma - i\omega} \to \frac{iNfe^2}{m\omega}$$

negligible

$$k^2 = \frac{\omega^2}{c^2} - \frac{\mu_0 N f e^2}{m\omega} = \frac{1}{c^2}\left(\omega^2 - \omega_p^2\right)$$

plasma freq

$$\omega_p = e\sqrt{\frac{Nf}{m\epsilon_0}} \qquad \frac{1}{c^2} = \mu_0 \epsilon_0$$

$$k^2 = \frac{1}{c^2}\left(\omega^2 - \omega_p^2\right)$$

$$\omega = \sqrt{k^2 c^2 + \omega_p^2} \qquad \frac{d\omega}{dk}$$

similar to Klein Gordon dispersion relation

$$E^2 = p^2c^2 + m^2c^4 \; : \; \hbar^2\omega^2 = \hbar^2 k^2 c^2 + m^2 c^4$$

$v_p >> c$ $\qquad$ $v_g < c$

case 1 for $\omega > \omega_p$ ↗

for $\omega < \omega_p$ $\quad k^2 = \frac{1}{c^2}(\omega^2 - \omega_p^2)$ ← negative

k is then imaginary

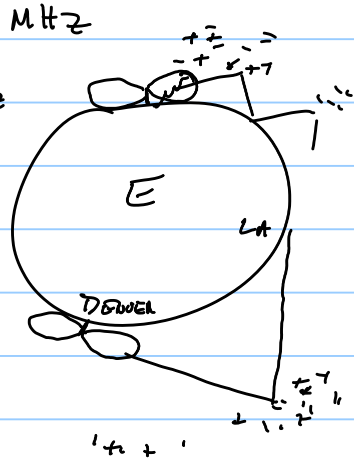
$$e^{i(kx-\omega t)} = e^{-|k|x}\, e^{-i\omega t}$$

↑ absorption

$N \simeq 10^{11} \frac{\text{electrons}}{m^3}$ $\quad \omega_p \simeq 3$ MHz

AM band < 3MHz

KOA 850 KHz



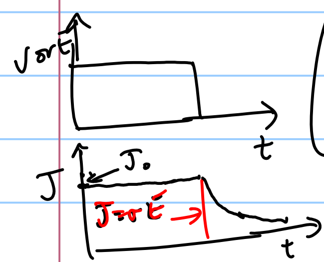
$$\tilde{y}(t) = \tilde{y}_0 e^{-i\omega t} = \frac{e/m}{\underbrace{\omega^2 + i\gamma\omega}} E_0 e^{-i\omega t} \qquad v = \frac{d\tilde{y}}{dt}$$

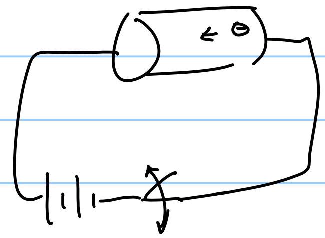
$$J = N(-e)\,v \qquad m\frac{d\vec{v}}{dt} = -\gamma v - eE_0\, e^{-i\omega t} \qquad \text{Newts 2nd}$$

Microscopic model $\Rightarrow$ $$\frac{d\vec{J}}{dt} = -\gamma\vec{J} - eE_0\, e^{-i\omega t}$$

turn applied field off quickly

$v = Ed$





$$-J_0\gamma e^{-\gamma t} + \gamma J_0 e^{-\gamma t}$$

$$\frac{dJ}{dt} + \gamma J = 0$$

$$J = J_0 e^{-\gamma t}$$

$$\frac{dJ}{dt} = -J_0\gamma e^{-\gamma t}$$