PHGN 462 Homework 4

1) (From Pollack and Stump 13.5)

Consider light traveling from $x = -\infty$, incident normally on a plate of glass with thickness a. The plate is parallel to the yz plane, with one face at x = 0 and the other at x = a. The index of refraction is $n_0 = 1$ for x < 0 and x > a, and n = 1.5 for $0 \le x \le a$. The electromagnetic field in the region x < 0 is a superposition of right and left traveling waves (where right means $\hat{\imath}$ and left means $-\hat{\imath}$), which are the incident and reflected waves. In the region $0 \le x \le a$ there are both right and left traveling waves, and in the region x > a there is only the transmitted right traveling wave.

a) Write **E**(x,t) and **B**(x,t) in the three regions, letting $\hat{\jmath}$ be the polarization direction. Write the four boundary conditions on the wave amplitudes.

b) Solve for the transmission coefficient T, i.e., the ratio of transmitted intensity to incident intensity.

c) Plot T as a function of *ka* (*k* times *a*), where *k* is the incident wave vector magnitude.

2) Read the Wikipedia article on dielectric mirrors, mirrors that work via principles of constructive and destructive interference using thin films. Feel free to follow up on links therein if you find them interesting. Then read the article on ellipsometry, which is a little bit more advanced and involves both polarization and phase. There's a lot of stuff here, so write up a paragraph or two regarding what you personally found interesting in the articles. Answers will vary a lot.

Also check out the dielectric mirrors offered by my personal favorite optics supplier, Thorlabs.

http://www.thorlabs.us/NewGroupPage9.cfm?ObjectGroup_ID=139&gclid=CNbTvs6_57ECFSIbQgodrTEACw

The above link leads to a page full of reflectivity vs. wavelength graphs demonstrating just what these mirrors can do. You can make a mirror to reflect just about anything, while also transmitting just about anything else. This comes in handy in laser optics, when you want to pass a laser beam of one color and reflect one of a different color.

3) (From Pollack and Stump 13.9)

a) From first principles, set up the boundary conditions for a plane wave incident normally on the surface of a conductor. Let the incident wave travel in the x direction, and be polarized in the y direction. The reflected and transmitted waves travel in the –x and +x directions, respectively. The wave vector κ in the conductor is complex, and the dispersion relation is:

$$\kappa^2 = +i\mu\sigma\omega + \mu\varepsilon\omega^2$$

Solve the boundary conditions, and show that the amplitude of the reflected wave is correctly given by:

$$\frac{E_0''}{E_0} = \frac{1-n_2}{1+n_2} = \frac{\omega - c\kappa}{\omega + c\kappa}$$

b) For a good conductor, i.e., $\sigma \gg \varepsilon\omega$, derive:

$$R \approx 1 - \frac{4\omega c\kappa_1}{c^2(\kappa_1^2 + \kappa_2^2)} = 1 - \sqrt{\frac{8\omega\varepsilon_0}{\sigma}}$$

c) Show that for a good conductor, at the surface the field of the reflected wave is approximately equal but opposite to the field of the incident wave. You may take the dispersion relation from part a as given. Also, you may assume the conductor is not significantly magnetic.

Note that this problem is largely done in Pollack and Stump for you if you'd like to check it out (chapter 13.3). Then it's just a matter of going through the steps yourself and filling in the gaps.

4) (From Pollack and Stump 13.12)

As an example of group velocity, consider a Gaussian wave pulse of some quantity $\phi(x,t)$ that undergoes wave motion, given by:

$$\phi(x,t) = \int_{-\infty}^{\infty} e^{i(kx-\omega t)} f(k) \frac{dk}{2\pi}$$

where $f(k) = f_0 e^{-(k-k_0)^2 a^2}$. The k range is $(-\infty, \infty)$, but the integrand is peaked at $k = k_0$, and the width of the peak is of order 1/a. Assume that within the peak $\omega(k)$ may be approximated by:

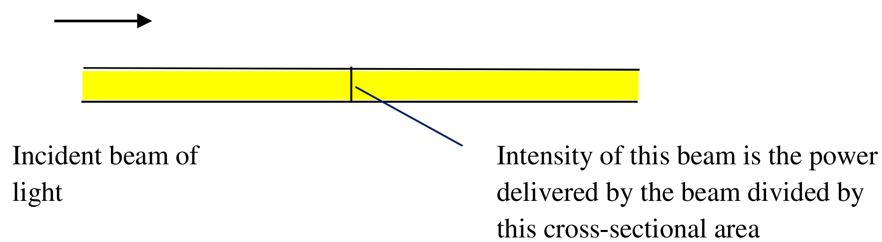
$$\omega(k) = \omega(k_0) + (k - k_0)\omega'(k_0)$$

a) Evaluate the integral, and obtain explicitly the function $\phi(x,t)$. (Hint: Let $k = k_0 + q$, change the variable of integration to q, and use a table of integrals or an analytic computer program (e.g. Mathematica or Maple) to evaluate the integral.)

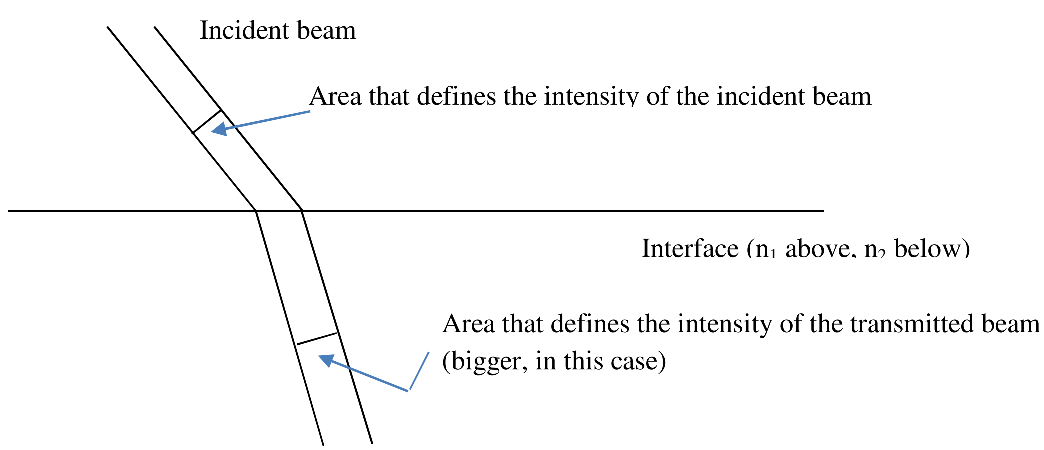
b) Show explicitly that the phase velocity is $\omega(k_0)/k_0$ and the group velocity is $\omega'(k_0)$.

Comments/hints: We're starting from a Fourier-domain representation of a wave pulse with a Gaussian envelope. Then we do the Fourier transform to get back to position space, using some math tricks in the process to give us an equation that lets us derive the expressions for group and phase velocities. Keep in mind that you can find a velocity by tracking a particular value of the argument of a function.

5) Non-normal incidence has one other tiny little complication we haven't talked about yet. So far we've been using the definitions $R = \frac{I_R}{I_I}$ and $T = \frac{I_T}{I_I}$, and power conservation has required that $\frac{I_R}{I_I} + \frac{I_T}{I_I} = 1$. But if you use the Fresnel equations to generate those intensities in the general, non-normal case, they don't add up to 1. That's bad. Power is still conserved in non-normal incidence problems. The issue is that intensity as we've defined it is power per unit area *assuming normal incidence*:



When light is incident on a surface at some angle, we still define intensity in the same way, but the width of a particular slice of wave *changes* as we go from incident to transmitted:



a) In order to preserve the core principle that power in equals power out, we need to use geometry to generalize the equation $\frac{I_R^2}{I_I^2} + \frac{I_T^2}{I_I^2} = 1$. Figure out how. You're going to need to add some trig factors here and there.

b) Using the above, show $r + t = 1$, where r and t are the new versions of R and T that properly respect the geometry of the non-normal case. To keep it simple, just do the TE case, and let $n_1 = 1$.