Homework 1 due 31 Aug. 2007 in class
PH462/507 EM Waves and Optical Physics posted: 24 August 2007

Reading: Heald and Marion (HM) ch4: 4.4-4.6, 4.8; Chen: ch1 (handout online)

Come to see me or email me if there are any questions on what is being asked. I'll post any clarifications or hints on the course website.

1) Consider the constant current in a wire described in HM 4-16, and discussed in class.
   a. Calculate the vector potential A(r) from the B-field, for r<a and r>a. Set the reference level at A(0) = 0.
   b. Verify that you get the same B from $\mathbf{B} = \nabla \times \mathbf{A}$.
   c. Verify that you can get the original current density distribution using eqn 4.58.

2) Use the Einstein summation technique to prove the vector identity:
   $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \bullet \mathbf{C}) - \mathbf{C}(\mathbf{A} \bullet \mathbf{B})$

3) HM 4.12 You should assume the time variation is sufficiently slow that the electric field is conservative, and that adding the potentials around the circuit gives zero (Kirchhoff's rule).

4) *Single-particle trajectories in plasmas*

Plasma physics is in general a complicated combination of electrodynamics and fluid mechanics. However, some of the effects can be investigated by considering the collisionless motion of individual charged particles in fields. The starting point is the Lorentz force equation.

a) A spatially-uniform, constant-in-time magnetic field $\mathbf{B} = B_0 \hat{\mathbf{z}}$ penetrates a plasma. Solve the equations of motion to show that the electrons with an initial velocity $\mathbf{v} = \mathrm{v}_0 \hat{\mathbf{x}}$ execute circular orbits. Calculate expressions for the frequency of revolution (the cyclotron frequency) and the radius of the orbit (the Larmor radius).
b) In a plasma, the electrons and ions are moving around in random directions. Suppose the field strength is sufficiently high so that the Larmor radius is much smaller than the dimensions of the plasma. Calculate the motion of electrons and ions (**r**(t)) with *arbitrary* initial direction. Can a magnetic field as described above be used to confine a plasma?
c) For order-of-magnitude estimates, we can assign a mean thermal velocity such that the kinetic energy of the particle is equal to the thermal energy $k_B T$. If the thermal energy is 100eV and the field strength is 1 Tesla, calculate the cyclotron frequency and the Larmor radius for electrons and protons. You will need to convert the expressions to SI for this.
d) Returning to the situation of part (a), where there is a known initial particle direction, suppose we add a constant electric field such that $\mathbf{E} \cdot \mathbf{B} = 0$. Describe the orientation and magnitude of the electric field such that the particle moves without acceleration.

5) *Plasma frequency*

In class we discussed how a plasma has a natural oscillation frequency, the plasma frequency. Imagine starting with a block of plasma with electron and ion number densities equal (singly ionized, overall neutral).

a) Use Gauss' law to calculate the restoring force for particles when the collection of electrons is moved to the side by a distance Δx. Then show that the motion is harmonic with the frequency $\omega_p = \sqrt{4\pi\, n_e e^2 / m_e}$.

b) Calculate ω_p, the Debye length λ_d, and the number of particles in a "Debye sphere" $N_D \equiv n_e \frac{4}{3}\pi\lambda_D^3$ for the following parameters:

   a. A glow discharge: with $n_e = 10^{10}\text{cm}^{-3}$, $k_B T_e = 2$ eV.
   b. The earth's ionosphere: with $n_e = 10^{6}\text{cm}^{-3}$, $k_B T_e = 0.1$ eV.
   c. A θ-pinch device: with $n_e = 10^{17}\text{cm}^{-3}$, $k_B T_e = 800$ eV. A θ-pinch is a fusion plasma confinement device based in part on problem 5.

6) HM 4-23