Dipole Calc

PHGN361 Exam 2: NAME

1. A water molecule with dipole moment $\vec{p}_0$ pointing along the z axis is located at the origin. A nitrogen atom with polarizability α_0 is located on the z axis a distance D from the origin. Derive an expression for the dipole moment of the nitrogen molecule.

$$\vec{E} = \frac{P_0}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})\Big|_{\theta=0} = \frac{2P_0}{4\pi\epsilon_0 r^3}\hat{r}$$

5 pts

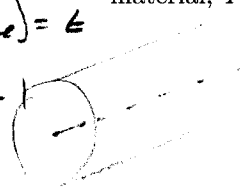
$$\vec{P} = \alpha\vec{E} = \frac{2P_0\alpha}{4\pi\epsilon_0 r^3}\hat{r}$$

5 pts

2. Charge, distributed on a line with charge density λ_0 Coulombs per meter, is embedded at the center of a cylinder of linear material (with susceptibility χ_e and radius R). Find the electric field in the material, $\vec{P}$, and the bound surface and volume charge densities.

$\epsilon_0(1+\chi_e) = \epsilon$

$\chi_e = \frac{\epsilon}{\epsilon_0} - 1$



$$\oint \vec{D}\cdot d\vec{a} = Q_f \Rightarrow D\,2\pi r\ell = \lambda\ell \qquad D = \frac{\lambda}{2\pi r} = \epsilon E$$

3 pts

$$E = \frac{\lambda}{2\pi\epsilon r} \qquad P = \epsilon_0\chi_e E = \epsilon_0\chi_e\frac{\lambda}{2\pi\epsilon r} = \frac{\epsilon_0\chi_e}{2\pi\epsilon_0(1+\chi_e)r} = \frac{\beta}{r}$$

3 pts

$$-\vec{\nabla}\cdot\vec{P} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\beta}{r}\right) = 0 = \rho_b$$

2 pts

$$\sigma_b = \vec{P}\cdot\hat{n} = \frac{\beta}{R}$$

2 pts

3. Semi-infinite conducting planes $\phi = 0$ and $\phi = \pi/6$ are separated by an infinitesimal insulating gap as shown. Let $V(\phi = 0) = 0$ and $V(\phi = \pi/6) = 100$ Volts. Assume that V depends only on ϕ and use cylindrical coordinates (z axis at the gap and perpendicular to the page).

points

10 (a) Find both $\vec{E}$ and V in the region between the plates.

3 (b) Now that the problem is solved, why can you say that the assumption that V depends only on ϕ was correct?

3 (c) Find the capacitance for two conducting plates each of 1 m^2 area if the plates are in the geometry of the semi-infinite conducting planes. Assume that the field is that given in part (a) and that the plates are separated by a gap width of 4 mm.

Please put your work on the back page.