MATH 225 - Differential Equations September 16, 2008

Project 1, Fall 2008 **Due**: September 24, 2008

Lab 1.1 - Rate of Memorization Model

For this project we consider lab 1.1 of Differential Equations pages 144 to 145. After reading this material construct a lab report addressing each of the following questions:[1]

1. Assuming that the rate of learning is proportional to the amount left to be learned we establish the following model equation,

$$\frac{dL}{dt} = k\,(1 - L)\,, \quad k \in \mathbb{R}^{+}. \tag{1}$$

   (a) Construct a list of variables and parameters associated with (1) and describe the meaning of each.

   (b) Analytically solve (1) using the methods discussed in section 1.2 of the text.

   (c) Discuss qualitative behavior of the solutions to (1) through the equation's:

      i. Equilibrium Points

      ii. Phase Line

   (d) Complete question one of this lab on page 144.

2. Given a mathematical model it is common to use recorded data to estimate values for unknown parameters within the model. In is not always obvious how one can best fit the parameters of a continuous problem to discrete data. There is inherent error both in the continuous approximation and data acquisition methods.

   (a) For this problem try to estimate your personal $k-$value in various ways.

      i. Using the model and considering the derivative $\dfrac{dL}{dt}$ as a finite difference $\dfrac{\triangle L}{\triangle t} = \dfrac{L_2 - L_1}{t_2 - t_1}$ where $\triangle t$ is assumed to be one-minute, approximate your personal $k-$value in terms of known data.

      ii. Using your analytic solution from 1(b) approximate k using your calculated data.

      iii. Using a graph of an Euler's method solution, pick your personal $k-$value to fit your model to the data compiled in 1(d).

   (b) Using your different approximations for your personal $k-$values estimate how long it would take to learn a list of 50 and 100 three-digit numbers. Compare and contrast the results of created by your approximate $k-$values.

   Hint: Now knowing all 20 numbers does not correspond to $L = 1$ but $L = 2/5$ or $L = 2/10$. Use this to recompute your two $k-$values from part (a).

3. In a short essay format summarize your results from the previous questions. After your summary address the following questions:

   - Did you feel that your personal $k-$values gave an accurate description of your rate of learning? Be sure to justify your conclusion with the data from your experiments.
   - Did you feel that this mathematical model gives an accurate description of your learning process? In particular are there scales where your mathematical model was a good approximation?

   Based on your response to these questions explain where you feel the model has room for improvement. Be sure to specifically address the mathematical model and how it would change to incorporate these improvements.

[1] Your report should be well organized and clearly presented. If steps are unclear then include more steps or make annotations clarifying the procedure and purpose. Be sure to label and title any included graphs or tables of data.